

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI  
(END SEMESTER EXAMINATION)**

CLASS: IMSc  
BRANCH: MATHEMATICS AND COMPUTING

SEMESTER : I  
SESSION: MO/2022

SUBJECT: MA109 MATRIX THEORY

TIME: 3 Hours

FULL MARKS: 50

**INSTRUCTIONS:**

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
  2. Attempt all questions.
  3. The missing data, if any, may be assumed suitably.
  4. Before attempting the question paper, be sure that you have got the correct question paper.
  5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
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- |  | CO      | BL  |
|--|---------|-----|
| Q.1(a) Determine whether the matrix $A = \begin{bmatrix} 0 & 3 & 2 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is nilpotent.   | [5] CO2 | BT4 |
| Q.1(b) Show that if $A$ is a $2 \times 2$ orthogonal matrix then each of the two rows is a unit vector and the dot product of the two rows equals zero.  | [5] CO1 | BT4 |
| Q.2(a) Find the reduced row echelon form of the matrix $\begin{bmatrix} 5 & 20 & -18 \\ 3 & 12 & -14 \\ -4 & -16 & 13 \end{bmatrix}$   | [5] CO1 | BT1 |
| Q.2(b) Find the rank of the matrix $\begin{bmatrix} -2 & 1 & 1 & 15 \\ 6 & -1 & -2 & -36 \\ 1 & -1 & -1 & -11 \\ -5 & -5 & -5 & -14 \end{bmatrix}$   | [5] CO1 | BT1 |
| Q.3(a) Solve the following system of linear equations by Gauss Elimination method:<br>$4x - 2y - 7z = 5, -6x + 5y + 10z = -11, -2x + 3y + 4z = -3, -3x + 2y + 5z = -5$                           | [5] CO1 | BT3 |
| Q.3(b) Find the matrix of the linear transformation $T: R^3 \rightarrow R^3$ with respect to the standard ordered basis for $R^3$ :<br>$T(x, y, z) = (-6x + 4y - z, -2x + 3y - 5z, 3x - y + 7z)$ | [5] CO1 | BT1 |
| Q.4(a) Determine whether the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is diagonalizable.   | [5] CO2 | BT4 |
| Q.4(b) Find the minimal polynomial for the matrix $\begin{bmatrix} 3 & 3 & 0 \\ 3 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$  | [5] CO1 | BT1 |
| Q.5(a) Find $A^{-2}$ for the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ using Cayley Hamilton Theorem.   | [5] CO1 | BT1 |
| Q.5(b) Determine the nature of quadratic form generated by the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$   | [5] CO2 | BT4 |