

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION MO/SP20**)

CLASS: BTECH / IMSc.
BRANCH: ALL / FT

SEMESTER : I
SESSION : MO/22

SUBJECT: MA103 MATHEMATICS - I

TIME: 02 HOURS

FULL MARKS: 25

INSTRUCTIONS:

1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates

- | | | CO | BL |
|--|-----|-----|----|
| Q.1(a) Find whether the sequence $\{a_n\}$ is monotonically increasing, bounded and convergent or not. $a_n = \frac{3n}{n+1}$ for all $n \in \mathbb{N}$ | [2] | CO1 | 1 |
| Q.1(b) Test the convergence of the series $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \dots$ | [3] | CO1 | 5 |
| Q.2(a) Find whether the following series is convergent or not $\sum_{n=1}^{\infty} \frac{(n - \ln n)^n}{2^n \cdot n^n}$ | [2] | CO1 | 1 |
| Q.2(b) Test the convergence of the series $1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \dots (x > 0)$ | [3] | CO2 | 5 |
| Q.3(a) $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & b \end{bmatrix}$. Find for which value of b, rank of A is 3 | [2] | CO2 | 2 |
| Q.3(b) Find for which real value of b the following equations have a non zero solution $x + 2y + 3z = bx$, $3x + y + 2z = by$ and $2x + 3y + z = bz$ | [3] | CO2 | 2 |
| Q.4(a) Can the vector (7, 7, 9, 11) be expressed as a linear combination of vectors (2, 0, 3, 1), (4, 1, 3, 2) and (1, 3, -1, 3)? If, so, find the linear combination. | [2] | CO2 | 3 |
| Q.4(b) Applying Cayley Hamiltonian theorem find A^{-1} . $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ | [3] | CO2 | 3 |
| Q.5(a) Evaluate whether following limit exists or not. $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^4}{x^2 + y^8}$ | [2] | CO3 | 1 |
| Q.5(b) If $u = x^y$; check whether $\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial x \partial y \partial x}$ or not. | [3] | CO2 | 3 |