BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: BTECH/IMSC **SEMESTER : I**
BRANCH: ALL/FT **SESSION : MO/2022**

SUBJECT: MA103 MATHEMATICS - I

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:

1. **The question paper contains 5 questions each of 10 marks and total 50 marks.**
2. **Attempt all questions.**
3. **The missing data, if any, may be assumed suitably.**
4. **Before attempting the question paper, be sure that you have got the correct question paper.**
5. **Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.**

			CO	BL
Q.1(a)	Test the convergence of the series: $\frac{x}{1.2}+\frac{x^2}{3.4}+\frac{x^3}{5.6}+\ldots\ldots,\ x>0.$	[5]	1	4
Q.1(b)	Test for absolute and conditional convergence of the series: $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^p},p>0$	[5]	1	4
Q.2(a)	Find the eigen values and corresponding eigen vectors of the following matrix: $\begin{pmatrix}3 & 1 & -1\\ -2 & 1 & 2\\ 0 & 1 & 2\end{pmatrix}$	[5]	2	3
Q.2(b)	Using Cayley Hamilton theorem find the inverse of the matrix $A=\begin{bmatrix}1 & 2\\ -1 & 3\end{bmatrix}$ and express $A^6-4A^5+8A^4-12A^3+14A^2$ as a linear polynomial of A.	[5]	2	4
Q.3(a)	Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial x}+z\frac{\partial u}{\partial z}+3\tan u=0$ when $u=\sin^{-1}\left(\frac{x+2y+z}{\sqrt{x^8+y^8+z^8}}\right)$.	[5]	3	2
Q.3(b)	Discuss the maximum and minimum of the function: $$f(x,y)=x^3+3xy^2-15x^2-15y^2+72x$$	[5]	3	2
Q.4(a)	Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{2xy}(x^2+y^2)\,dzdydx$ by changing coordinate system.	[5]	4	4
Q.4(b)	Evaluate $\iint_R (x+y)^2\,dxdy,$ where R is the parallelogram in the xy-plane with vertices (1,0), (3,1), (2,2), (0,1) using the transformation u = x + y and v = x- 2y.	[5]	4	4
Q.5(a)	Find the Divergence and Curl of of the vector function $\vec{F}(x,y,z)=e^{xyz}(xy^2\hat{i}+yz^2\hat{j}+zx^2\hat{k})$ at the point (1, 2, 3).	[5]	5	3
Q.5(b)	Applying Green's theorem, evaluate $\oint_C (x^5+3y)\,dx+(2x-e^{y^3})dy,$ where C is the circle $(x-1)^2+(y-5)^2=4.$	[5]	5	4

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