CLASS:   IMSc                                              SEMESTER: I
BRANCH:  MATHEMATICS & COMPUTING                           SESSION: MO/2022

**SUBJECT: MA102 REAL ANALYSIS**

TIME:   2 HOURS                                            FULL MARKS: 25

**INSTRUCTIONS:**
1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates.

-----------------------------------------------------------------------------------------------------------------

|  |  |  | CO | Bl |
|---|---|---|---|---|
| Q.1(a) | Define supremum and infimum of a non-empty subset of $\mathbb{R}$. | [2] | 1 | 1 |
| Q.1(b) | Calculate the supremum and the infimum of the set $\left\{\frac{n+(-1)^{2n}}{n}, n \in \mathbb{N}\right\}$. | [3] | 2 | 3 |
| Q.2(a) | Construct examples of a closed set and an open set which are not intervals. | [2] | 1 | 3 |
| Q.2(b) | Let $G \subseteq \mathbb{R}$ be an open set and $F \subseteq \mathbb{R}$ be a closed set. Explain the nature of the set $G \cap F^c$? | [3] | 2 | 2 |
| Q.3(a) | Find the formula for the nth term of the sequence $\frac{100}{2}, \frac{-(100)^2}{6}, \frac{(100)^3}{24}, \frac{-(100)^4}{120}, \dots$ | [2] | 1 | 3 |
| Q.3(b) | Examine the convergence or divergence of the sequence $\left\{\frac{n!}{n^n}\right\}$. | [3] | 3 | 4 |
| Q.4(a) | Examine the monotonicity of the sequence $\left\{\frac{4^{n+1}+3^n}{4^n}\right\}$. | [2] | 1 | 4 |
| Q.4(b) | Show that every Cauchy sequence is bounded. | [3] | 2 | 4 |
| Q.5(a) | Find the limit of the sequence $\left\{\frac{1+\sqrt[2]{2}+\sqrt[3]{3}+\dots\sqrt[n]{n}}{n}\right\}$. | [2] | 3 | 3 |
| Q.5(b) | Examine convergence or divergence of the series $\sum_{n=1}^{\infty}\frac{1}{n}$. | [3] | 3 | 4 |

:::::: 17/01/2023 ::::::M