BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc **SEMESTER : I**
BRANCH: MATHEMATICS & COMPUTING **SESSION : MO/2022**

SUBJECT: MA102 REAL ANALYSIS

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Before attempting the question paper, be sure that you have got the correct question paper.**
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Find the Supremum and the Infimum of the set A= $\left\{\frac{1-(-1)^n}{n^2}; n \in \mathbb{N}\right\}$.	[5]	1	1
Q.1(b)	Define a limit point and an interior point of a nonempty subset of $\mathbb{R}$. Calculate limit points and interior points of the set $\mathbb{Q}$ of all rational numbers, if there be any.	[5]	1	1+3
Q.2(a)	Check the monotonicity, boundedness and convergence of the sequence $a_n = \frac{n!}{n^n}, n \in \mathbb{N}$. If convergent, find its limit.	[5]	3	1
Q.2(b)	Show that every Cauchy sequence is bounded.	[5]	2	3
Q.3(a)	Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2-n+1}$.	[5]	3	4
Q.3(b)	Examine whether the following series is convergent or not $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2^n}$.	[5]	3	4
Q.4(a)	For each $n \in \mathbb{N}, let\ f_n: [0,\infty) \to [0,\infty)\ be\ defined\ by\ f_n(x) = \frac{nx}{1+n^2x^2}$. Find a function $f: [0,\infty) \to [0,\infty)$ such that $\{f_n\}$ converges to f on $[0,\infty)$. Explain if the convergence is uniform	[5]	3	2
Q.4(b)	Show that the series $\sum_{n=1}^{\infty} \frac{Sin\, nx}{n^2}$ is uniformly convergent over $\mathbb{R}$.	[5]	3	3
Q.5(a)	A function f is defined on [0,1] by $f(x) = 1$, if x is rational; 0, if x is irrational. Examine whether f is R −integrable on [0,1].	[5]	3	4
Q.5(b)	Apply the fundamental theorem of integral calculus to find the value of $\int_{-2}^{2} f(x)dx$, where $f(x) = 3x^2 Cos\frac{\pi}{x^2} + 2\pi Sin\frac{\pi}{x^2}, x \neq 0,$ $= 0, x = 0.$	[5]	5	3

::::::16/03/2023::::::M