CLASS: IMSC   SEMESTER: I
BRANCH: IMH   SESSION: MO/2022

SUBJECT: MA101 CALCULUS - I

TIME: 2 HOURS   FULL MARKS: 25

INSTRUCTIONS:
1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates

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		CO	BL

Q.1(a)   Expand $e^{a\sin x}$ in power of x [2] CO1

Q.1(b)   If $y = \tan^{-1}\left(\frac{a+x}{a-x}\right)$, prove that $(a^2 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + n(n + 1)y_n = 0$ [3] CO1

Q.2(a)   Verify the mean value theorem for the function $f(x) = \frac{|x|}{x}$, $x \in [-1, 1]$, [2] CO1

Q.2(b)   Find the points of inflexion and the intervals of convexity and concavity of the curve [3] CO1
$y = e^{-x^2}$

Q.3(a)   Find the radius of curvature at P (1,4) on the rectangular hyperbola with equation [2] CO2
$xy = 4$

Q.3(b)   Show that the parabola $y = -x^2 + x + 1$ and $x = -y^2 + y + 1 = 0$ have the [3] CO2
same circle of curvature at the point (1, 1)

Q.4(a)   Show that $f(x, y) = \frac{xy}{\sqrt{x^2+y^2}}$ when $(x, y) \neq (0, 0)$ [2] CO3
$\qquad = 0 \qquad$ when $(x, y) = (0, 0)$
is continuous at (0, 0)

Q.4(b)   Find the asymptotes of the curve $y^3 + x^2y + 2xy^2 - y + 1 = 0$ [3] CO2

Q.5(a)   If $u - \log(\tan x + \tan y + \tan z)$, then find the value of [2] CO3
$(\sin 2x)\frac{\partial u}{\partial x} + (\sin 2y)\frac{\partial u}{\partial y} + (\sin 2z)\frac{\partial u}{\partial z}$

Q.5(b)   If $f(x, y) = (x^2 + y^2)\tan^{-1}\frac{y}{x}$ when $x \neq 0$ and $f(0, y) = \frac{\pi y}{2}$, show that [3] CO3
$f_{xy}(0,0) \neq f_{yx}(0, 0)$