BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI (END SEMESTER EXAMINATION)

CLASS: **IMSC SEMESTER: I BRANCH:** MATHS & COMP. SESSION: MO/2022 SUBJECT: MA101 CALCULUS - I TIME: **FULL MARKS: 50** 3 Hours **INSTRUCTIONS:** 1. The question paper contains 5 questions each of 10 marks and total 50 marks. 2. Attempt all questions. 3. The missing data, if any, may be assumed suitably. 4. Before attempting the question paper, be sure that you have got the correct question paper. 5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall. CO BL Q.1(a) If $y = \frac{\sinh^{-1}x}{\sqrt{1+x^2}}$ show that [5] $(1+x^2)y_{n+2} + (2n+3)xy_{n+1} + (n+1)^2y_n = 0$ Q.1(b) Verify Rolle's theorem for the function [5] 1 $f(x) = 2x^3 + x^2 - 4x - 2$ Q.2(a) Find the asymptotes of the curve [5] 2 $x^3 - 2x^2y + xy^2 + x^2 - xy + 2 = 0$ Q.2(b) Find the radius of curvature at the point θ on the curve [5] 2 $x = 3a\cos\theta - a\cos 3\theta$, $y = 3a\sin\theta - a\sin 3\theta$ Q.3(a) If $x^2 + y^2 + u^2 - v^2 = 0$ and uv + xy = 0, find $\frac{\partial(u,v)}{\partial(x,y)}$ 3 [5] Q.3(b) Find the extreme values of [5] 3 $u = x^2v^2 - 5x^2 - 8xv - 5v^2$ Q.4(a) Evaluate using the concept of Beta and Gamma function [5] 4 $\int\limits_{3}^{4} \sqrt{(x-3)(7-x)} \, dx$ Evaluate the integral Q.4(b)[5] 4 $\int_{0}^{\infty} e^{-x} \frac{\sin \alpha x}{x} dx$ Q.5(a) Determine the area bounded between the curves $y = x^2 + 1$ and y = 7 - x[5] 5 O.5(b) Find the volume of the solid produced by the revolution of the upper half of the loop [5] 5 of the curve $y^2 = x^2(2 - x)$ about the x-axis.

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