

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC
BRANCH: QEDS

SEMESTER : III
SESSION : MO/2022

SUBJECT: ED201 DIFFERENTIAL EQUATIONS

TIME: 3:00 Hours

FULL MARKS: 50

INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
 2. Attempt all questions.
 3. The missing data, if any, may be assumed suitably.
 4. Before attempting the question paper, be sure that you have got the correct question paper.
 5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
-

- CO
- Q1 (a) Find the general solution of the following differential equation
- $$(D^2 + 2)y = x^2 e^{3x} + e^x \cos 2x, \quad \text{where } D \equiv \frac{d}{dx}. \quad [5] \quad \text{CO2}$$
- (b) Using appropriate transformation, reduce the following equation into a linear differential equation with constant coefficients, and hence solve the equation.
- $$(5 + 2x)^2 y'' - 6(5 + 2x)y' + 8y = 8(5 + 2x)^2. \quad [5] \quad \text{CO2}$$
- Q2 (a) Show that $\frac{1}{3x^3y^3}$ is an integrating factor of the differential equation $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$, and hence solve it. [3] CO1
- (b) Solve the differential equation $(D^2 - 1)y = 1$, given that $y(0) = 0$ and y tends to a finite limit as $x \rightarrow -\infty$. [2] CO1
- (c) Using Laplace transform, evaluate the integral $\int_0^\infty \frac{e^{-t} \sin t}{t} dt$. [5] CO4
- Q3 (a) Evaluate the power series solution of the following differential equation about $x = 0$.
- $$(1 - x^2)y'' - 2xy' + 2y = 0. \quad [7] \quad \text{CO3}$$
- (b) Classify the ordinary point, regular singular point, and irregular singular point at $x = -2$ for the following differential equation [3] CO3
- $$x(x - 1)^2(x + 2)y'' + x^2y' - (x^3 + 2x - 1)y = 0.$$
- Q4 (a) Given that $f(t + 2\pi) = f(t)$. Find the Laplace transform of the function [4] CO4
- $$f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi. \end{cases}$$
- (b) Using Laplace transform, solve the following initial value problem. [6] CO4
- $$y'' - 3y' + 2y = e^{-t}, y(0) = 1, y'(0) = 0.$$
- Q5 Solve the following system of homogeneous linear differential equations by finding eigenvalue and eigenvectors of the coefficient matrix. [10] CO5
- $$\begin{pmatrix} y_1'(x) \\ y_2'(x) \\ y_3'(x) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}.$$