

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION)**

**CLASS: B.TECH.
BRANCH: ECE**

**SEMESTER: III
SESSION: MO/2022**

SUBJECT: EC213 PROBABILITY AND RANDOM PROCESSES

TIME: 2 HOURS

FULL MARKS: 25

INSTRUCTIONS:

1. The total marks of the questions are 25.
2. Candidates attempt for all 25 marks.
3. Before attempting the question paper, be sure that you have got the correct question paper.
4. The missing data, if any, may be assumed suitably.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

		CO	BL
Q1 (a)	If $A = \{2 \leq x \leq 5\}$ and $B = \{3 \leq x \leq 6\}$, then find $(A \cup B)(\overline{AB})$.	[2]	1 I
Q1 (b)	Trains X and Y arrive at a station at random between 8 A.M. and 8.20 A.M. Train X stops for four minutes and train Y stops for five minutes. Assuming that the trains arrive independently of each other, determine the probability that train X arrives before train Y.	[3]	1 V
Q2 (a)	Simplify, $\overline{A \cup B} \cup \overline{A \cup B}$	[2]	1 III
Q2 (b)	Box 1 contains 1000 bulbs of which 10% are defective. Box 2 contains 2000 bulbs of which 5% are defective. Two bulbs are picked from a randomly selected box. Assuming that both are defective, find the probability that they came from box 1.	[3]	1 I
Q3 (a)	Define probability density function of Gamma Distribution of random variable 'X' and find its cumulative distribution function.	[2]	2 I
Q3 (b)	If X is a Gaussian random variable with zero mean and unity variance, then show that the probability density function of random variable $Y = X^2$ is given by $f_Y(y) = \frac{1}{\sqrt{2\pi y}} e^{-\frac{y}{2}} U(y)$.	[3]	2 II
Q4 (a)	Estimate the value of $E[X/X \geq 1]$, if X is uniform distribution function in the range of [2, 4].	[2]	2 VI
Q4 (b)	The characteristics function of a random variable X is given by $\varphi_X(\omega) = \frac{1}{(1-j2\omega)^2}$. Find the mean and second moment of X.	[3]	2 I
Q5 (a)	Find 'k' if joint density function of the bivariate random variable (X, Y) is given by $f_{X,Y}(x,y) = \begin{cases} k(1-x)(1-y) & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$	[2]	3 I
Q5 (b)	If the probability density function is given by $f_{X,Y}(x,y) = \begin{cases} \frac{6}{5}(x+y^2) & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$, then determine the probability, $P\left(\frac{1}{4} \leq y \leq \frac{3}{4}\right)$	[3]	3 I