BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: BTECH | **SEMESTER : III**
BRANCH: ECE | **SESSION : MO/2022**

SUBJECT: EC213 PROBABILITY AND RANDOM PROCESSES

TIME: 3:00 Hours | **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a)	For three events A, B and C, show that $P(AB/C) = P(A/BC)P(B/C)$ and $P(ABC) = P(A/BC)\,P(B/C)P(C)$.	[2]	I	II
Q.1(b)	For three independent events A, B and C, if one or more of these events are replaced by their complements, then show that the resulting events are also independent.	[3]	I	II
Q.1(c)	Three switches, S_1, S_2, and S_3 connected in parallel operates independently. Each switch remains closed with probability p. Find the probability that switch S_1 is open given that an input signal is received at the output.	[5]	I	I
Q.2(a)	Explain the memoryless property of exponential distribution.	[2]	II	II
Q.2(b)	Explain the Poisson Distribution of random variable X and evaluate its mean and variance.	[3]	II	V
Q.2(c)	If $Y = \sqrt{X}$, and X is an exponential random variable, show that Y represents a Rayleigh random variable. Further, find the first order moment of Y.	[5]	II	II
Q.3(a)	Define covariance and correlation coefficient of two random variables X and Y. Write the conditions when two random variables are uncorrelated and orthogonal.	[2]	III	I
Q.3(b)	For the random variables X and Y jointly normal, explain the joint density function $f_{XY}(x, y)$ and evaluate its marginal densities $f_X(x)$ and $f_Y(y)$.	[3]	III	V
Q.3(c)	Given, $f_{XY}(x, y) = \begin{cases} k & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$, determine $f_{X/Y}(x/y)$	[5]	III	V
Q.4(a)	Define and prove the Chebyshev inequality.	[2]	IV	I
Q.4(b)	Explain stochastic convergence of a random sequence. Further, explain and compare the various types of stochastic convergences.	[3]	IV	II
Q.4(c)	A coin is weighted so that its probability of landing on heads is 20 %. Suppose the coin is flipped 20 times. Estimate the bound using Markov and Chebyshev inequalities for the probability it lands on heads at least 16 times.	[5]	IV	VI
Q.5(a)	Define wide sense stationary (WSS) processes. What is WSS white noise process?	[2]	V	I
Q.5(b)	Suppose that $X(t)$ is a WSS process with autocorrelation $R(\tau) = e^{-\alpha\|\tau\|}$. Determine the second moment of the random variable $X(3) - X(5)$.	[3]	V	V
Q.5(c)	The mean and autocorrelation function of the input stochastic process $X(t)$ of a linear system with the impulse response $h(t)$ are given as $\eta_X(t)$ and $R_{XX}(t_1, t_2)$ respectively. Determine the mean and autocorrelation of output random process $y(t)$ of the system.	[5]	V	V

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