BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: BE **SEMESTER : VII**
BRANCH: ECE **SESSION : MO/19**

SUBJECT: MEC1011 PROBABILITY MODELS & STOCHASTIC PROCESSES

TIME: 3.00Hrs. **FULL MARKS: 60**

INSTRUCTIONS:
1. The question paper contains 7 questions each of 12 marks and total 84 marks.
2. Candidates may attempt any 5 questions maximum of 60 marks.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) Define random variable and its distribution function. [2]

Q.1(b) The continuous random variable with density function $f(x) = \begin{cases} \frac{x}{9} + K & 0 \le x \le 6 \\ 0 & otherwise \end{cases}$. Find $P(2 \le x \le 5)$ [4]

Q.1(c) If the probability density of a random variable is given by $f_X(x) = K(1 - x^2)\ \ 0 < x < 1$. Find the value of K and $F_X(x)$. [6]

Q.2(a) Explain central limit theorem with example. [2]

Q.2(b) Consider that a pdf of a random variable X is $f_X\,(x) = \begin{cases} \frac{1}{K} & -2 \le x \le 3 \\ 0 & otherwise \end{cases}$. Another random variable is defined as Y=2X. Find (a) value of K (b) E(Y). [4]

Q.2(c) Given the function $\boldsymbol{f_{X,Y}(x,y) = \begin{cases} b(x+y)^2 & -2 < x < 2 \ and \ -3 < y < 3 \\ 0 & otherwise \end{cases}}$. Find the constant b such that this is a valid joint density function. Determine the marginal density function $\boldsymbol{f_X(x)}$ and $\boldsymbol{f_Y(y)}$. [6]

Q.3(a) Explain Jointly Gaussian random variables. [2]

Q.3(b) Discuss the sequences of random variables and its convergence. [4]

Q.3(c) The joint density function for X and Y is $f_{X,Y}(x,y) = \begin{cases} \frac{xy}{9} & 0 < x < 2 \ and \ 0 < y < 3 \\ 0 & otherwise \end{cases}$. Show that X and Y are uncorrelated. [6]

Q.4(a) Explain sampling theory in stochastic process. [2]

Q.4(b) The joint density functions of two random variables is given by [4]
$\boldsymbol{f_{X,Y}\,(x,y) = \frac{18y^2}{x^3} \ \ for \ 2 < x < \infty \ and \ 0 < y < 1}$. Find (a) E[X] (b) E[Y].

Q.4(c) A random process defined as X(t)= A coswt + B sinwt where A and B are uncorrelated, zero mean random variables having the variance σ^2 . Find (a) autocorrelation function (b) show that X(t) is wide sense stationary. [6]

Q.5(a) Explain Thermal noise. [2]

Q.5(b) The power spectral density of X(t) is given by $\boldsymbol{S_{XX}(w) = \begin{cases} 1 + w^2 \ for \ |w| < 1 \\ 0 \quad otherwise \end{cases}}$. Find the autocorrelation function. [4]

Q.5(c) Discuss the Markov process and Wiener process with examples. [6]

Q.6(a) Describe Linear filtering of stochastic process. [2]

Q.6(b) Explain autoregressive model of stochastic process. [4]

Q.6(c) X(t) is a stationary random process with spectral density $S_{XX}(w)$. Y(t) is another independent random process, $Y(t) = Acos\,(w_c t + \theta)$, where θ is a random variable uniformly distributed over $(-\pi, \pi)$. Find the spectral density function of Z(t)=X(t)Y(t). [6]

Q.7(a) Explain spectral estimation using AR model. [2]

Q.7(b) Explain Kalman filter with example. [4]

Q.7(c) Define an optimum filter? Derive the expression for a Wiener filter as an optimum filter. [6]

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