## BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI

(END SEMESTER EXAMINATION)

| CLASS: | MTECH |
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| BRANCH: | CSE/IT |

SEMESTER : III
SESSION : MO/19
SUBJECT: CS601 GRAPH THEORY
TIME: $\quad 3$ HOURS

## INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
Q.1(a) Prove that the number of vertices of odd degree in a graph is always even.
Q.1(b) Justify that a graph G is disconnected if and only if its vertex set can be partitioned into two nonempty disjoint sets V1 and V2 such that there exists no edge in G whose one end is in subset V1 and the other in subset V2.
Q.2(a) Write a short note on (i) Fundamental cutsets (ii) Spanning trees. Explain with examples.
Q.2(b) Consider the spanning tree $\{a, b, d, f\}$ (marked by curves on edges), in the given graph (Figure 1). Identify all the chords, and for each chord write down the fundamental circuit associated with it.


Figure 1
Q.3(a) Prove that a connected planar graph with $n$ vertices and e edges has e-n+2 regions.
Q.3(b) What do you mean by geometric dual of a graph? Explain with an example.
Q.4(a) What is an adjacency matrix? Write the adjacency matrix $X$ for the given graph (Figure 2).
Q.4(b) Compute $X^{2}$ for the given graph. Justify the value of $X^{2}[3,5]$ (that is, the entry in 3rd row and 5th column) in terms of the graph.


Figure 2
Q.5(a) What are independent sets? Given maximal independent sets, how can you find the chromatic number for a graph. Illustrate with an example.
Q.5(b) Prove that a graph of $n$ vertices is a complete graph if and only if its chromatic polynomial is $P_{n}(\lambda)=\lambda(\lambda$ 1) $(\lambda-2) \ldots . .(\lambda-n+1)$

