BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: M.TECH **SEMESTER : I**
BRANCH: CHEM **SESSION : MO/19**

SUBJECT: CL502 ADVANCED MATHEMATICAL TECHNIQUES FOR CHEMICAL ENGINEERING
TIME: 3:00 HOURS **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) Define Metric, Norm and Inner Product of vector with example. [5]

Q.1(b) Generate an orthonormal set from the linearly independent set (2,0,1), (2,1,3) and (4,1,2) in R^3 [5]

Q.2(a) The following equations is obtained after a finite difference technique is applied to a problem [5]

$$\frac{3}{2}x_1 - \frac{1}{8}x_2 - \frac{1}{8}x_3 = 1$$
$$-\frac{1}{8}x_1 + \frac{3}{2}x_2 - \frac{1}{8}x_4 = 1$$
$$-\frac{1}{8}x_1 + \frac{3}{2}x_3 - \frac{1}{8}x_4 = 1$$
$$-\frac{1}{8}x_2 - \frac{1}{8}x_3 + \frac{3}{2}x_4 = 2$$

If k=1 choice for $x^T = [1\ 1\ 1\ 1\ 1]$ use the Gauss-Siedel method to solve these linear simultaneous equations. Do three iterations only.

Q.2(b) Determine the eigenvalues and eigenvectors for the A using polynomial method. [5]

$$A = \begin{bmatrix} 2 & 2 & -6 \\ 2 & -1 & -3 \\ -2 & -1 & 1 \end{bmatrix}$$

Q.3(a) Classify partial differential equations based on coefficient and give example for each type. [5]

Q.3(b) Obtain the dominant eigenvalue and corresponding eigenvectors of the following matrix A using the power method. Also obtain the correct sign of the eigenvalue. [5]

$$A = \begin{bmatrix} 0 & 2 & 3 \\ -10 & -1 & 2 \\ -2 & 4 & 7 \end{bmatrix}$$

Q.4 Consider the problem of heat conduction in a one-dimensional slab(0 < x < 1), the surface at x = 0 is insulated and at x = 1 is losing heat to the environment at a rate proportional to the temperature difference between the surface and the ambient. The transient temperature profile is governed by [10]

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$$

Solve the partial differential equation using separation variable method.

Q.5 Solve the following problem using Green's function [10]

$$\frac{d^2T}{dx^2} - \frac{dT}{dt} = x$$

Subject to $T(x=0) = 1$, $\frac{dT}{dt}(x=1) = 2$

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