

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI  
(END SEMESTER EXAMINATION)**

CLASS: BE  
BRANCH: ECE

SEMESTER : VII  
SESSION : MO/18

SUBJECT: MEC1011 PROBABILITY MODELS & STOCHASTIC PROCESSES

TIME: 3:00 HRS.

FULL MARKS: 60

**INSTRUCTIONS:**

1. The question paper contains 7 questions each of 12 marks and total 84 marks.
  2. Candidates may attempt any 5 questions maximum of 60 marks.
  3. The missing data, if any, may be assumed suitably.
  4. Before attempting the question paper, be sure that you have got the correct question paper.
  5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
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- Q.1(a) What is random variable? Define conditional probability and conditional expectation. [2]
- Q.1(b) In an experiment the random variable  $X$  has the elements  $X = \{-3, -2, -1, 0, 1, 2\}$  with probabilities  $P_X(x) = \{0.2, 0.5k, k, 0.1, 0.3k, k\}$ . Find the value of  $k$  and sketch the distribution function  $F_X(x)$ . [4]
- Q.1(c) Given a probability density function  $f_X(x) = ae^{-b|x|}$  where  $-\infty \leq x \leq \infty$ . Find i) the Distribution function ii) relation between  $a$  and  $b$  iii)  $P(1 \leq X \leq 2)$ . [6]
- Q.2(a) Explain the central limit theorem with suitable example. [2]
- Q.2(b) A random variable  $X$  has exponential pdf given as  $f_X(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ . Evaluate the mean and variance of  $X$ . [4]
- Q.2(c) The joint pdf of bivariate random variable  $(X, Y)$  is given as  $f_{X,Y}(x, y) = \begin{cases} Kxy, & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$ . Find the value of  $K$  and determine whether  $X$  and  $Y$  are independent. [6]
- Q.3(a) What is random process? Describe the strict sense stationary and weak sense stationary process. [2]
- Q.3(b) Describe about sampling theory. Define the sample mean and sample variance. [4]
- Q.3(c) The random variables  $X$  and  $Y$  have the joint density function  $f_{X,Y}(x, y) = \frac{1}{24}, 0 < x < 6; 0 < y < 4$ . What is the expected value of the function  $g(x, y) = (XY)^2$  [6]
- Q.4(a) Explain ergodic random process with suitable example. [2]
- Q.4(b) Derive an expression for power spectral density. Describe the white noise. [4]
- Q.4(c) A stationary random process has an autocorrelation function given as  $R(\tau) = \frac{25\tau^2 + 36}{6.25\tau^2 + 4}$ . Find the mean, mean square value and variance of the process. [6]
- Q.5(a) Describe the Poisson process. [2]
- Q.5(b) The spectral density of a random process  $X(t)$  is given by  $S_{XX}(w) = \frac{w^2}{w^4 + 13w^2 + 36}$ . Find the autocorrelation function and the average power. [4]
- Q.5(c) A random process is defined as  $X(t) = A \cos(w_c t + \theta)$  where  $\theta$  is a uniform random variable over  $(0, 2\pi)$ . Verify whether the process is ergodic in the mean sense and autocorrelation sense. [6]
- Q.6(a) Discuss what happen to PSD of a random signal if we pass it through a linear system. [2]
- Q.6(b) Explain the thermal noise and shot noise. [4]
- Q.6(c) Describe Auto regressive process and find the relationship between autocorrelation sequence with model parameters. [6]
- Q.7(a) Explain Kalman filter with suitable example. [2]
- Q.7(b) Discuss on the spectral estimation using auto regressive model. [4]
- Q.7(c) What is an optimum filter? Derive the expression for a Wiener filter. [6]