BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: BE **SEMESTER : III**
BRANCH: IT **SESSION : MO/18**

SUBJECT: IT3021-DISCRETE MATHEMATICS AND GRAPH THEORY

TIME: 03:00 **FULL MARKS: 60**

INSTRUCTIONS:
1. The question paper contains 7 questions each of 12 marks and total 84 marks.
2. Candidates may attempt any 5 questions maximum of 60 marks.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) Let *p*: Babu is rich, *q*: Babu is happy. Give a simple verbal sentence which describes each of the following proposition: [4]
(i) $p \vee \sim q$ (ii) $\sim p \rightarrow q$ (iii) $\sim \sim p$ (iv) $(\sim p \wedge q) \rightarrow p$

(b) Investigate the following compound proposition for a truth table as a tautology: [8]
$$(p \wedge q) \vee (p \wedge r)$$

Q.2(a) Using principle of mathematical induction, show that [6]
$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{n}} > \sqrt{n}, \quad \text{for } n \geq 2 .$$

(b) Let $N = n^2 + n + 41$. Show that there are some values of n for which N is a prime number, and others for which it is not. It follows that there is no inductive step which would show that $n^2 + n + 41$ is a prime number for all possible n. [6]

Q.3(a) Find the generating function of the Fibonacci sequence $\{a_n\}$ defined by [6]
$$a_n = a_{n-1} + a_{n-2} \; ; \; a_0 = 0 , \; a_1 = 1 .$$

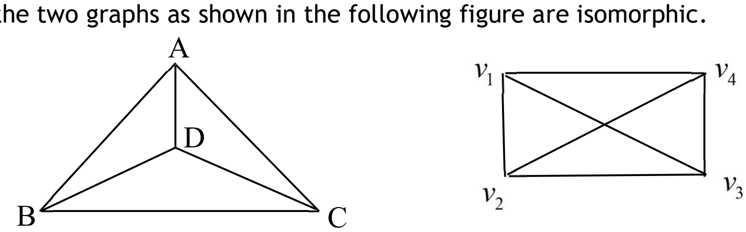
(b) Solve the recurrence relation $a_r = 4a_{r-1} - 4a_{r-2} + 4^r$, $r \geq 2$ using the generating function method, under the initial conditions $a_0 = 2$ and $a_1 = 8$. [6]

Q.4(a) Prove that the maximum number of edges in a simple disconnected graph *G* with *n* vertices and *k* components is $\frac{(n-k)(n-k+1)}{2}$. [8]

(b) Let G be a simple graph with 12 edges. If G has 6 vertices of degree 3 and the rest of the vertices have degree less than 3. Determine the (i) minimum number of vertices and (ii) maximum number of vertices. [4]

Q.5(a) Define Euler and Hamiltonian graphs with vivid description citing its construction. [8]
(b) In a graph, prove that an Euler path can also be a Hamiltonian path in some cases. Investigate it. If true, give an example of such case. [4]

Q.6(a) Show that the two graphs as shown in the following figure are isomorphic. [8]



(b) For each of the following degree sequences, determine if there exists a graph. Draw the graph. [4]
(i) $(5, 5, 4, 3, 2, 1)$ (ii) $(5, 4, 3, 2, 1, 1)$

Q.7 Write short notes of the following: [12]
(i) Labelled Trees (ii) Tree Searching (iii) Spanning Trees

********26.11.18******E**