

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: M.TECH
BRANCH: ECE

SEMESTER : I
SESSION : MO/18

SUBJECT: EC512 STOCHASTIC PROCESSES AND INFORMATION THEORY
TIME: 3 HOURS

FULL MARKS: 50

INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
 2. Attempt all questions.
 3. The missing data, if any, may be assumed suitably.
 4. Before attempting the question paper, be sure that you have got the correct question paper.
 5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
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- Q.1(a) State and derive an expression for power spectral density and its properties. [5]
Q.1(b) Explain wide sense stationary random processes. State the condition for two random vectors X and Y are independent and uncorrelated. [5]

- Q.2(a) Define entropy for continuous and discrete random variable. A binary memory less system produces two messages with probability p and 1-p. Show that the entropy is maximum when both messages are equiprobable. [5]
Q.2(b) Consider a BSC with $P(x_1) = \alpha$ [5]
(a) Prove that the mutual Information $I(X;Y)$ is given by
$$I(X;Y) = H(Y) + p \log_2(p) + (1-p) \log_2(1-p).$$

(b) Compute $I(X;Y)$ for $\alpha=0.5$ and $p=0.1$
(c) Repeat (b) for $\alpha=0.5$ and $p=0.5$ and Comment on the result.

- Q.3(a) Prove that the information capacity of a continuous channel of bandwidth W hertz, perturbed by additive Gaussian noise of power spectral density $N_0/2$ is given by $C = W \log_2 \left(1 + \frac{P}{N_0 W} \right)$ bits/sec. [5]

- Q.3(b) State and Explain Source Coding and Channel Coding theorem. [5]

- Q.4(a) Explain the importance of rate distortion theory. [5]
Q.4(b) Derive the expression of rate distortion function for Gaussian as well as Binary symmetric source and comments over the sketch of rate distortion function for both sources. [5]

- Q.5(a) For the AWGN multiple access channel, prove, using typical sequences, the achievability of any rate pairs (R_1, R_2) satisfying [5]

$$\begin{aligned} R_1 &< \frac{1}{2} \log \left(1 + \frac{P_1}{N} \right), \\ R_2 &< \frac{1}{2} \log \left(1 + \frac{P_2}{N} \right), \\ R_1 + R_2 &< \frac{1}{2} \log \left(1 + \frac{P_1 + P_2}{N} \right). \end{aligned}$$

- Q.5(b) Write short notes on Broadcast Channel and Relay Channel. [5]