BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION SP2024)

CLASS: IMSc. **SEMESTER: II**
BRANCH: QEDS **SESSION: SP/2024**

SUBJECT: ED111 INTERMEDIATE ANALYSIS

TIME: 03 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates
6. All the notations used in the question paper have usual meanings.

	Question	Marks	CO	BL
Q.1(a)	Let $\{f_n(x)\}$ be a sequence of function defined on $\mathbb{R}$ such that $f_n(x) = \frac{nx}{1+n^2x^2}$. Show that $\{f_n(x)\}$ is NOT uniformly convergent in any interval containing zero.	[5]	CO1	1,2
Q.1(b)	Determine the radius of convergence of the power series $\sum_{n=1}^{\infty}\left(n^{\frac{1}{3}}+1\right)^n (x-1)^n$.	[5]	CO2	3
Q.2(a)	Expand $(x+x^2)$ in Fourier series on $-\pi < x < \pi$.	[5]	CO2	3
Q.2(b)	Check the Riemann integrability of the function $f(x) = \begin{cases} \frac{1}{2^n}, \frac{1}{2^{n+1}} < x \le \frac{1}{2^n}, n = 0,1,2,3,\dots \\ 0, x = 0, \end{cases}$ on $[0,1]$. If integrable, find the value of $\int_0^1 f(x)dx$.	[5]	CO3	4
Q.3(a)	Show that the function $f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, \text{ if } x^2+y^2 \neq 0 \\ 0, \text{if } x = y = 0 \end{cases}$ possesses first order partial derivatives but is not differentiable at origin.	[5]	CO4	4
Q.3(b)	State Euler's theorem for the homogeneous function. Apply the theorem for the function $u = x^3y^2\sin^{-1}\left(\frac{y}{x}\right)$ to find the value of $xu_x + yu_y$.	[5]	CO4	5
Q.4(a)	Calculate Jacobian of x, y, z with respect to u, v, w when $u = x+y+z$, $uv = y+z, uvw = z$.	[5]	CO4	5
Q.4(b)	Find the volume of the solid bounded by the coordinate planes $x = 0, y = 0, z = 0$ and the surface $\left(\frac{x}{a}\right)^{\frac{1}{2}} + \left(\frac{y}{b}\right)^{\frac{1}{2}} + \left(\frac{z}{c}\right)^{\frac{1}{2}} = 1$.	[5]	CO5	
Q.5(a)	Evaluate the integral $\int\int_R e^{x^2}dx\,dy$, where R is a region bounded by $x = 2y, x = 2, y = 0$ and $y = 1$.	[5]	CO5	6
Q.5(b)	Find the value of the integral $\int\int_R \sqrt{x^2+y^2}\,dxdy$, the field of integration being R, the region in $xy-$plane bounded the circles $x^2+y^2 = 1$ and $x^2+y^2 = 4$ by transforming the integral into polar co-ordinates.	[5]	CO5	6

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