BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: BE **SEMESTER : III**
BRANCH: CSE **SESSION : MO/18**

SUBJECT: CS4101-DISCRETE MATHEMATICS STRUCTURE

TIME: 03:00 **FULL MARKS: 60**

INSTRUCTIONS:
1. The question paper contains 7 questions each of 12 marks and total 84 marks.
2. Candidates may attempt any 5 questions maximum of 60 marks.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) Suppose the predicate $F(x, y, t)$ is used to represent the statement that person x can fool person y at time t. Write the meaning of the formula $\forall x \exists y \exists t(\sim F(x, y, t))$ in words. [2]

(b) Write the following statements in symbolic form: [4]
(i) "If my computations are correct and I pay the electric bill, then I will run out of money. If I don't pay the electric bill, the power will be turned off. Therefore, if I don't run out of money and the power is still on, then my computations are incorrect." Convert this argument into logical notations using the variables $\boldsymbol{c}, \boldsymbol{b}, \boldsymbol{r}, \boldsymbol{p}$ for propositions of computations, electric bills, out of money and the power respectively.
(ii) The sum of two positive integers is always positive.

(c) Prove or Disprove: $[(p \wedge q) \rightarrow r] \rightarrow [\sim r \rightarrow (\sim p \wedge \sim q)]$ is a tautology. [6]

Q.2(a) Prove that, for any integers x, y, the product xy is even if and only if either x is even or y is even. [4]

(b) Prove that the following expression is true for every natural numbers $\boldsymbol{n}$, [8]

$$\mathbf{1}.\boldsymbol{n} + \mathbf{2}.(\boldsymbol{n}-\mathbf{1}) + \mathbf{3}.(\boldsymbol{n}-\mathbf{2}) + \cdots + (\boldsymbol{n}-\mathbf{1}).\mathbf{2} + \boldsymbol{n}.\mathbf{1} = \frac{\mathbf{1}}{\mathbf{6}}\boldsymbol{n}(\boldsymbol{n}+\mathbf{1})(\boldsymbol{n}+\mathbf{2})$$

Q.3(a) Prove that, if f is $o(g)$ (little-o) then f is $O(g)$ (Big-O). [4]

(b) Prove that, if a and r are real numbers and $r \neq 0$, then [4]

$$\sum_{j=0}^{n} ar^j = \begin{cases} \dfrac{ar^{n+1} - a}{r-1} & if\ r \neq 1 \\ (n+1)a & if\ r = 1 \end{cases}$$

(c) Show that $\log_a x \in o(x)$ where a is a positive number different from 1. [4]

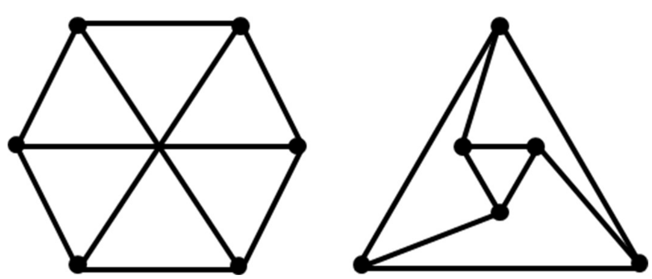
Q.4(a) Show that the "greater than or equal" relation ($\geq$) is a partial ordering on the set of integers. [6]

(b) Find the reflexive, symmetric and transitive closures of the following relation [6]

$$R = \{(1,1), (1,2), (2,2), (2,4), (3,2), (4,1)\}.$$

Q.5(a) Prove that, if a (p, q) graph G is k-connected then $q \geq \frac{pk}{2}$ [2]

(b) Draw the complements of the following two graphs. Are these complements isomorphic to each other? [4]



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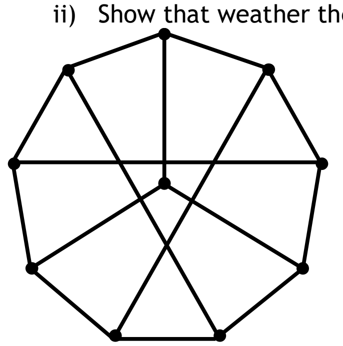
(c) i) Is the complete bipartite graph $K_{7,10}$ Hamiltonian? Justify your answer. [6]

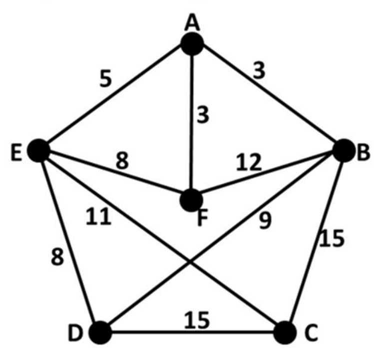
ii) Show that weather the given graph is Hamiltonian or not. Justify your answer.



Q.6(a) Find the no. of vertices of degree 1 in a binary tree. [2]

(b) Let the tree T has 50 edges: the removal of certain edges from T yields two disjoint trees T_1 and T_2 such that the number of vertices in T_1 equals the number of edges of T_2. Determine the number of edges and vertices of T_1 and T_2. [4]

(c) Using Kruskal's Algorithm find a minimal spanning tree of the weighted graph given below [6]



Q.7(a) Prove that order of the subgroup divides the order of the group. [4]

(b) If $(G_1,*_1)$ and $(G_2,*_2)$ are groups, then show that $G = G_1 \times G_2$ i.e., $(G,*)$ is a group with binary operation $*$ defined by $(a_1, b_1) * (a_2, b_2) = (a_1 *_1 b_1, a_2 *_2 b_2)$. [8]

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