CHAPTER 9

DEPRECIATION

An analysis of costs and profits for any business operation requires recognition of the fact that physical assets decrease in value with age. This decrease in value may be due to physical deterioration, technological advances, economic changes, or other factors which ultimately will cause retirement of the property. The reduction in value due to any of these causes is a measure of the ***depreciation.?*** The economic function of depreciation, therefore, can be employed as a means of distributing the original expense for a physical asset over the period during which the asset is in use.

Because the engineer thinks of depreciation as a measure of the decrease in value of property with time, depreciation can immediately be considered from a cost viewpoint. For example, suppose a piece of equipment had been put into use 10 years ago at a total cost of $31,000. The equipment is now worn out and is worth only $1000 as scrap material. The decrease in value during the lo-year period is $30,000; however, the engineer recognizes that this $30,000 is in reality a cost incurred for the use of the equipment. This depreciation cost was spread over a period of 10 years, and sound economic procedure would require part of this cost to be charged during each of the years. The application

†According to the Internal Revenue Service, depreciation is defined as "A reasonable allowance for the exhaustion, wear, and tear of property used in the trade or business including a reasonable allowance for obsolescence." The terms ***amortization*** and ***depreciation*** are often used interchangeably. Amortization is usually associated with a definite period of cost distribution, while depreciation usually deals with an unknown or estimated period over which the asset costs are distributed. Depreciation and amortization are of particular significance as an accounting concept which serves to reduce taxes.

of depreciation in engineering design, accounting, and tax studies is almost always based on costs prorated throughout the life of the property.

Meaning of Value

From the viewpoint of the design engineer, the total cost due to depreciation is the original or new value of a property minus the value of the same property at the end of the depreciation period. The original value is usually taken as the total cost of the property at the time it is ready for initial use. In engineering design practice, the total depreciation period is ordinarily assumed to be the length of the property's useful life, and the value at the end of the useful life is assumed to be the probable scrap or salvage value of the components making up the particular property.

It should be noted here that the engineer cannot wait until the end of the depreciation period to determine the depreciation costs. These costs must be prorated throughout the entire life of the property, and they must be included as an operating charge incurred during each year. The property value at the end of the depreciation period and the total length of the depreciation period cannot be known with certainty when the initial yearly costs are determined. Consequently, it is necessary to estimate the final value of the property as well as its useful life. In estimating property life, the various factors which may affect the useful-life period, such as wear and tear, economic changes, or possible technological advances, should be taken into consideration.

When depreciation is not used in a prorated-cost sense, various meanings can be attached to the word value. One of these meanings involves appraisal of both initial and final values on the basis of conditions at a certain time. The difference between the estimated cost of new equivalent property and the appraised value of the present asset is ***known*** as the ***appraised depreciation.*** This concept involves determination of the values of two assets at one date as compared with the engineering-cost concept, which requires determination of the value of one asset at two different times.

Purpose of Depreciation as a Cost

Consideration of depreciation as a cost permits realistic evaluation of profits earned by a company and, therefore, provides a basis for determination of Federal income taxes. Simultaneously, the consideration of depreciation as a cost provides a means whereby funds are set aside regularly to provide recovery of the invested capital. When accountants deal with depreciation, they must follow certain rules which are established by the U.S. Bureau of Internal Revenue for determination of income taxes. These rules deal with allowable life for the depreciable equipment and acceptable mathematical procedures for allocating the depreciation cost over the life of the asset.

Although any procedure for depreciation accounting can be adopted for internal company evaluations, it is highly desirable to keep away from the necessity of maintaining two sets of accounting books. Therefore, the engineer should be familiar with Federal regulations relative to depreciation and should follow these regulations as closely as possible in evaluating depreciation as a cost.

TYPES OF DEPRECIATION

The causes of depreciation may be physical or functional. ***Physical depreciation*** is the term given to the measure of the decrease in value due to changes in the physical aspects of the property. Wear and tear, corrosion, accidents, and deterioration due to age or the elements are all causes of physical depreciation. With this type of depreciation, the serviceability of the property is reduced because of physical changes. Depreciation due to all other causes is known as ***functional depreciation.***

One common type of functional depreciation is ***obsolescence.*** This is caused by technological advances or developments which make an existing property obsolete. Even though the property has suffered no physical change, its economic serviceability is reduced because it is inferior to improved types of similar assets that have been made available through advancements in technology.

Other causes of functional depreciation could be (1) change in demand for the service rendered by the property, such as a decrease in the demand for the product involved because of saturation of the market, (2) shift of population center, (3) changes in requirements of public authority, (4) inadequacy or insufficient capacity for the service required, (5) termination of the need for the type of service rendered, and (6) abandonment of the enterprise. Although some of these situations may be completely unrelated to the property itself, it is convenient to group them all under the heading of functional depreciation.

Because depreciation is measured by decrease in value, it is necessary to consider all possible causes when determining depreciation. Physical losses are easier to evaluate than functional losses, but both of these must be taken into account in order to make fair allowances for depreciation.

Depletion

Capacity loss due to materials actually consumed is measured as ***depletion.*** Depletion cost equals the initial cost times the ratio of amount of material used to original amount of material purchased. This type of depreciation is particularly applicable to natural resources, such as stands of timber or mineral and oil deposits.

Costs for Maintenance and Repairs

The term ***maintenance*** conveys the idea of constantly keeping a property in good condition; ***repairs*** connotes the replacing or mending of broken or worn parts of a property. The costs for maintenance and repairs are direct operating expenses which must be paid from income, and these costs should not be confused with depreciation costs.

The extent of maintenance and repairs may have an effect on depreciation cost, because the useful life of any property ought to be increased if it is kept in good condition. However, a definite distinction should always be made between costs for depreciation and costs for maintenance and repairs.

SERVICE LIFE

The period during which the use of a property is economically feasible is known as the service *life* of the property. Roth physical and functional depreciation are taken into consideration in determining service life, and, as used in this book, the term is synonymous with ***economic*** or ***useful*** life. In estimating the probable service life, it is assumed that a reasonable amount of maintenance and repairs will be carried out at the expense of the property owner.

Many data are available concerning the probable life of various types of property. Manufacturing concerns, engineers, and the U.S. Internal Revenue Service (IRS) have compiled much information of this sort. All of these data are based on past records, and there is no certainty that future conditions will be unchanged. Nevertheless, by statistical analysis of the various data, it is possible to make fairly reliable estimates of service lives.

The U.S. Internal Revenue Service recognizes the importance of depreciation as a legitimate expense, and the IRS has issued formal statements which list recommended service lives for many types of properties.? Prior to July 12, 1962, Federal regulations for service lives and depreciation rates were based on the so-called ***Bulletin "F," "Income Tax Depreciation and Obsolescence-Estimated Useful Lives and Depreciation Rates"*** as originally published by the U.S. Internal Revenue Service in 1942. In July, 1962, the ***Bulletin "F"*** regulations were replaced by a set of new guidelines based on four groups of depreciable assets. The 1971 Revenue Act of the United States provided more flexibility in choosing depreciation life by allowing a choice of depreciation life of 20 percent longer or shorter than the guideline lives called for by earlier tax laws for machinery, equipment, or other assets put in service after December 31, 1970. This is known as the ***Class Life Asset Depreciation Range System*** (ADR).

Table 1 presents estimated service lives for equipment based on the four group guidelines as recommended by the Internal Revenue Service in the 1962 Federal regulations. These values, along with similar values as presented in ***Bulletin "F,"*** can serve as an indication of acceptable and useful lives to those not using other procedures.

†For an up-to-date presentation of Federal ***income-tax*** regulations as related to depreciation, including estimation of service lives, see the latest annual issue of "Prentice-Hall Federal Taxes," Prentice-Hall Information Services, Paramus, NJ 07652.

TABLE 1
Estimated life of equipment

The following tabulation for estimating the life of equipment in years is an abridgement of information from "Depreciation-Guidelines and Rules" (Rev. Proc. 62-21) issued by the Internal Revenue Service of the U.S. Treasury Department as Publication No. 456 (7-62) in July, 1962. See Table 2 for an extended and more flexible interpretation including repair allowance as approved by the Federal regulations in 1971.

	Life, years
Group I: General business assets	
1. Office furniture, fixtures, machines, equipment	10
2. Transportation	
a. Aircraft	6
b. Automobile	3
c. Buses	9
d. General-purpose trucks	4-6
e. Railroad cars (except for railroad companies)	15
f. Tractor units	4
g. Trailers	6
h. Water transportation equipment	18
3. Land and site improvements (not otherwise covered)	20
4. Buildings (apartments, banks, factories, hotels, stores, warehouses)	40-60
Group II: Nonmanufacturing activities (excluding transportation, communications, and public utilities)	
1. Agriculture	
a. Machinery and equipment	10
b. Animals	3-10
c. Trees and vines	variable
d. Farm buildings	25
2. Contract construction	
a. General	5
b. Marine	12
3. Fishing	variable
4. Logging and sawmilling	6-10
5. Mining (excluding petroleum refining and smelting and refining of minerals)	10
6. Recreation and amusement	10
7. Services to general public	10
8. Wholesale and retail trade	10
Group III: Manufacturing	
1. Aerospace industry	8
2. Apparel and textile products	9
3. Cement (excluding concrete products)	20
4. Chemicals and allied products	11
5. Electrical equipment	
a. Electrical equipment in general	12
b. Electronic equipment	8
6. Fabricated metal products	12
7. Food products, except grains, sugar and vegetable oil products	12
8. Glass products	14
9. Grain and grain-mill products	17

(Continued)

TABLE 1
Estimated life of equipment (Continued)

	Life, years
Group III: Manufacturing (continued)	
10. Knitwear and knit products	9
11. Leather products	11
12. Lumber, wood products, and furniture	10
13. Machinery unless otherwise listed	12
14. Metalworking machinery	12
15. Motor vehicles and parts	12
16. Paper and allied products	
a. Pulp and paper	16
b. Paper conversion	12
17. Petroleum and natural gas	
a. Contract drilling and field service	6
b. Company exploration, drilling, and production	14
c. Petroleum refining	16
d. Marketing	16
18. Plastic products	11
19. Primary metals	
a. Ferrous metals	18
b. Nonferrous metals	14
20. Printing and publishing	11
21. Scientific instruments, optical and clock manufacturing	12
22. Railroad transportation equipment	12
23. Rubber products	14
24. Ship and boat building	12
25. Stone and clay products	15
26. Sugar products	18
27. Textile mill products	12-14
28. Tobacco products	15
29. Vegetable oil products	18
30. Other manufacturing in general	12
Group IV: Transportation, communications, and public utilities	
1. Air transport	6
2. Central steam production and distribution	28
3. Electric utilities	
a. Hydraulic	50
b. Nuclear	20
c. Steam	28
d. Transmission and distribution	30
4. Gas utilities	
a. Distribution	35
b. Manufacture	30
c. Natural-gas production	14
d. Trunk pipelines and storage	22
5. Motor transport (freight)	8
6. Motor transport (passengers)	8
7. Pipeline transportation	22
8. Radio and television broadcasting	6

(Continued)

TABLE 1
Estimated life of equipment *(Continued)*

	Life, years
Group IV: Transportation, communications, and public utilities (continued)	
9. Railroads	
a. Machinery and equipment	14
b. Structures and similar improvements	30
c. Grading and other right of way improvements	variable
d. Wharves and docks	20
10. Telephone and telegraph communications	variable
11. Water transportation	20
12. Water utilities	50

Table 2 gives a partial listing of the *Class Life Asset Depreciation Range System* (CLADR) as recommended for use by Federal regulations in 1971. The table shows the basic guideline life period as recommended in the earlier regulations along with the 20 percent variation allowed plus recommended guidelines for repair and maintenance allowance. Although these values are recommended by the Internal Revenue Service, the IRS does not require taxpayers to use the indicated lives. However, if other life periods are used, the taxpayer must be prepared to support the claim.

Tax-law changes put into effect with the 1981 Economic Recovery Act and modified in 1986 have instituted a new system of depreciation known as the *Accelerated Cost Recovery System* (ACRS). The latter has replaced the former ADR system for most tangible depreciable property used in a trade or business placed in service on or after January 1, 1981. In the ACRS [or *Modified Accelerated Cost Recovery System* (MACRS) which went into effect for property put into service on or after January 1, 19871, the recovery of capital costs as depreciation was determined over statutory periods of time using statutory percentages depending on the *class life* of the property and the number of years since the property was placed in service. The statutory periods of time were generally shorter than the useful life of the asset or the period for which it was used to produce income.

The statutory class lives for the Modified Accelerated Cost Recovery System are as follows where the key factor is the ADR (Asset Depreciation Range) midpoint designation, which corresponds in general to the asset guideline period shown in Table 2:

Three-year class-ADR midpoint of 4 years and less. This includes items such as machinery and equipment used in research, some automobiles, and certain types of trailers.

Five-year class-ADR midpoint of 4 to 10 years. This includes most production machinery, heavy trucks, and some automobiles and light trucks.

TABLE 2
Class life asset depreciation range†

Description of class life asset	Asset depreciation range (ADR) (in years) Lower limit	Asset guideline period (Midpoint)	Upper limit	Annual asset guideline repair allowance, percentage of cost
Assets used in business activities:				
Office furniture, fixtures, and equipment	8	10	12	2
Information systems, computers, peripheral equipment	5	6	7	7.5
Data handling equipment, except computers	5	6	7	15
Airplanes, except commercial	5	6	7	14
Automobiles, taxis	2.5	3	3.5	16.5
Buses	7	9	11	11.5
Light general-purpose trucks	3	4	5	16.5
Heavy general-purpose trucks	5	6	7	10
Railroad cars and locomotives, except owned by railroad transportation companies	12	15	18	8
Tractor units for use over-the-road	3	4	5	16.5
Trailers and trailer-mounted containers	5	6	7	10
Vessels, barges, tugs and similar water-transportation equipment	14.5	18	21.5	6
Land improvements		20		
Industrial steam and electricity generation and/or distribution systems	22.5	33.5	33.5	2.5
Assets used in agriculture	8	10	12	11
Assets used in mining	8	10	12	6.5
Assets used in drilling of oil and gas wells	5	6	7	10
Assets used in exploration for and production of petroleum and natural gas deposits	5	6	7	10
Assets used in petroleum refining	13	16	19	7
Assets used in marketing of petroleum and petroleum products	13	16	19	4
Assets used in contract construction other than marine	4	5	6	12.5
Assets used in marine contract construction	9.5	12	14.5	5

† Values were excerpted from the listing given with full description of each category in the 1988 "Prentice Hall Federal Taxes" guide as updated from the original Federal regulation to the March 21, 1977 Revenue Procedure in the Internal Revenue Bulletin. The official documents originally setting up the ADR system were U.S. Treasury Decision 7128 in 1971 and Revenue Procedure 71-25 in 1971.

TABLE 2
Class life asset depreciation range† (*Continued*)

Description of class life asset	Asset depreciation range (ADR) (in years): Lower limit	Asset guideline period (Midpoint)	Upper limit	Annual asset guideline repair allowance, percentage of cost
Assets used in the manufacture of:				
Grain and grain-mill products	9.5	12	14.5	5
Sugar and sugar products	13.5	17	20.5	6
Vegetable oils and vegetable-oil products	14.5	18	21.5	4.5
Other food and kindred products	14.5	18	21.5	3.5
Tobacco and tobacco products	12	15	18	5
Knitted goods	6	7.5	9	7
Nonwoven fabrics	8	10	12	15
Wood products and furniture	5	6	7	10
Pulp and paper	8	10	12	6.5
Chemicals and allied products	9	11	13	5.5
Rubber products	11	14	17	5
Finished plastic products	9	11	13	5.5
Glass products	11	14	17	12
Cement	16	20	24	3
Machinery	8	10	12	11
Electrical equipment	9.5	12	14.5	5.5
Motor vehicles	9.5	12	14.5	9.5
Assets used in electric, gas, water, and steam utility services:				
Electric utility nuclear production plant	16	20	24	3
Electric utility steam production plant	22.5	28	33.5	5
Electric utility transmission and distribution plant	24	30	36	4.5
Gas utility distribution facilities	28	35	42	2
Gas utility manufactured gas production plant	24	30	36	2
Substitute natural gas-coal gasification (Lurgi process with advanced methanation)	14.5	18	21.5	15
Natural gas production plant	11	14	17	4.5
Liquefied natural gas plant	17.5	22	26.5	4.5
Water utilities	40	50	60	1.5
Central steam utility production and distribution	22.5	28	33.5	2.5

†Values were excerpted from the listing given with full description of each category in the 1988 "Prentice Hall Federal Taxes" guide as updated from the original Federal regulation to the March 21, 1977 Revenue Procedure in the Internal Revenue Bulletin. The official documents originally setting up the ADR system were U.S. Treasury Decision 7128 in 1971 and Revenue Procedure 71-25 in 1971.

Seven-year class-ADR midpoint of 10 to 16 years. Included here are items such as office furniture and equipment.

Ten-year class-ADR midpoint of 16 to 20 years. This includes properties such as tank cars and assets used in petroleum refining and food manufacturing.

Fifteen-year class-ADR midpoint of 20 to 25 years. Included here are items related to certain chemical production processes and some utilities.

Twenty-year class-ADR midpoint of 25 years or more. This includes many utilities and electrical distribution systems.

For the Accelerated Cost Recovery System in effect after 1980 and before 1987, the statutory classes were 3 year, 5 year, 10 year, and 15 year. The two classes of 7 year and 20 year were added in the Modified Accelerated Cost Recovery System for properties put into service on January 1, 1987 or later.

There has been considerable demand for a wider choice of service lives for properties, and the widespread revision and reinterpretation of the national income-tax laws in 1954, 1962, 1971, 1981, and 1986 met part of this demand. During times of national emergencies, the United States Congress may approve rapid-amortization policies to make it more attractive for concerns to invest in additional plants and equipment needed for the national welfare. Certificates of necessity can be obtained for certain types of industries, and these certificates permit writing off various percentages of the value of new equipment over selected periods of time.

SALVAGE VALUE

Salvage value is the net amount of money obtainable from the sale of used property over and above any charges involved in removal and sale. If a property is capable of further service, its salvage value may be high. This is not necessarily true, however, because other factors, such as location of the property, existing price levels, market supply and demand, and difficulty of dismantling, may have an effect. The term salvage *value* implies that the asset can give some type of further service and is worth more than merely its scrap or junk value.

If the property cannot be disposed of as a useful unit, it can often be dismantled and sold as junk to be used again as a manufacturing raw material. The profit obtainable from this type of disposal is known as the *scrap,* or *junk, value.*

Salvage value, scrap value, and service life are usually estimated on the basis of conditions at the time the property is put in use. These factors cannot be predicted with absolute accuracy, but improved estimates can be made as the property increases in age. It is advisable, therefore, to make new estimates from time to time during the service life and make any necessary adjustments in the depreciation costs. Because of the difficulties involved in making reliable estimates of salvage and scrap values, engineers often neglect the small error involved and designate these values as zero. Federal tax regulations generally limit salvage or scrap values to 10 percent or less of the initial value of the property.

PRESENT VALUE

The *present value* of an asset may be defined as the value of the asset in its condition at the time of valuation. There are several different types of present values, and the standard meanings of the various types should be distinguished.

Book Value, or Unamortized Cost

The difference between the original cost of a property, and all the depreciation charges made to date is defined as the *book value* (sometimes called *unamortized cost*). It represents the worth of the property as shown on the owner's accounting records.

Market Value

The price which could be obtained for an asset if it were placed on sale in the open market is designated as the *market value*. The use of this term conveys the idea that the asset is in good condition and that a buyer is readily available.

Replacement Value

The cost necessary to replace an existing property at any given time with one at least equally capable of rendering the same service is known as the *replacement value*.

It is difficult to predict future market values or replacement values with a high degree of accuracy because of fluctuations in market demand and price conditions.† On the other hand, a future book value can be predicted with absolute accuracy as long as a constant method for determining depreciation costs is used. It is quite possible for the market value, replacement value, and book value of a property to be widely different from one another because of unrealistic depreciation allowances or changes in economic and technological factors.

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†See Chap. 11 (Optimum Design and Design Strategy) for a discussion on inflation and the strategy for considering it.

METHODS FOR DETERMINING DEPRECIATION

Depreciation costs can be determined by a number of different methods, and the design engineer should understand the bases for the various methods. The Federal government has definite rules and regulations concerning the manner in which depreciation costs may be determined. These regulations must be followed for income-tax purposes as well as to obtain most types of governmental support. Since the methods approved by the government are based on sound economic procedures, most industrial concerns use one of the government-sanctioned methods for determining depreciation costs, both for income-tax calculations and for reporting the concern's costs and profits.† It is necessary, therefore, that the design engineer keep abreast of current changes in governmental regulations regarding depreciation allowances.

In general, depreciation accounting methods may be divided into two classes: (1) arbitrary methods giving no consideration to interest costs, and (2) methods taking into account interest on the investment. Straight-line, declining-balance, and sum-of-the-years-digits methods are included in the first class, while the second class includes the sinking-fund and the present-worth methods.

Straight-Line Method

In the straight-line *method* for determining depreciation, it is assumed that the value of the property decreases linearly with time. Equal amounts are charged for depreciation each year throughout the entire service life of the property. The annual depreciation cost may be expressed in equation form as follows:

$$d = \frac{V - V_s}{n} \tag{1}$$

where d = annual depreciation, \$/year
V = original value of the property at start of the service-life period, completely installed and ready for use, dollars
V_s = salvage value of property at end of service life, dollars
n = service life, years

The asset value (or book value) of the equipment at any time during the service life may be determined from the following equation:

$$V_a = V - a\,d \tag{2}$$

†An alternate procedure often used by industrial concerns is to use straight-line depreciation for reporting profits and one of the accelerated-depreciation methods as approved by Federal regulations for income-tax calculations.

where V_a = asset or book value, dollars, and a = the number of years in actual use.

Because of its simplicity, the straight-line method is widely used for determining depreciation costs. In general, design engineers report economic evaluations on the basis of straight-line depreciation unless there is some specific reason for using one of the other methods.

Because it is impossible to estimate exact service lives and salvage values when a property is first put into use, it is sometimes desirable to reestimate these factors from time to time during the life period of the property. If this is done, straight-line depreciation can be assumed during each of the periods, and the overall method is known as *multiple straight-line depreciation.* Figure 9-1 shows how the asset value of a property varies with time using the straight-line and the multiple straight-line methods for determining depreciation.

The straight-line method may be applied on the basis of units of production or predicted amount of service output, instead of life years. The depreciation may be based on miles, gallons, tons, number of unit pieces produced, or other measures of service output. This so-called *unit-of-production* or *service-output* method is particularly applicable when depletion occurs, as in the exploitation of natural resources. It should also be considered for properties having useful lives that are more dependent on the number of operations performed than on calendar time.

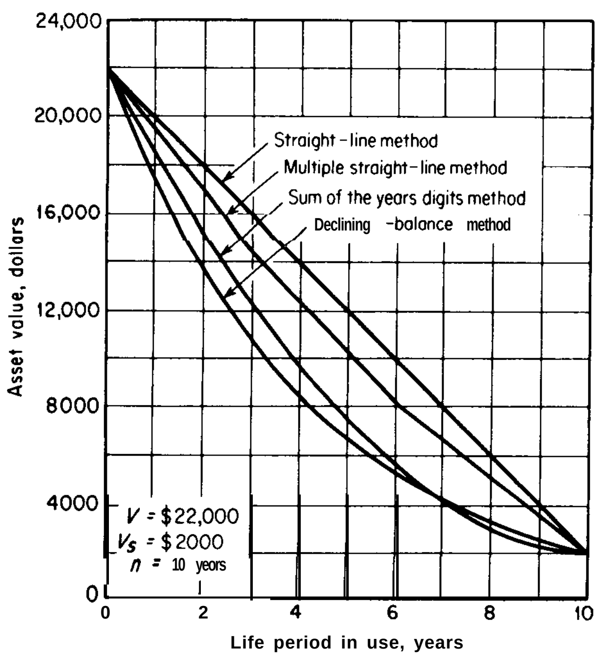

FIGURE 9-1
Comparison of straight-line, multiple straight-line, sum-of-the-years-digits, and declining-balance methods for determining depreciation.

Declining-Balance (or Fixed Percentage) Method

When the declining-balance method is used, the annual depreciation cost is a fixed percentage of the property value at the beginning of the particular year. The fixed-percentage (or declining-balance) factor remains constant throughout the entire service life of the property, while the annual cost for depreciation is different each year. Under these conditions, the depreciation cost for the first year of the property's life is $V\mathfrak{f}$, where $\mathfrak{f}$ represents the fixed-percentage factor.

At the end of the first year

$$\text{Asset value} = V_a = V(1 - \mathfrak{f}) \tag{3}$$

At the end of the second year

$$V_a = V(1 - \mathfrak{f})^2 \tag{4}$$

At the end of a years

$$V_a = V(1 - \mathfrak{f})^a \tag{5}$$

At the end of n years (i.e., at the end of service life)

$$V_a = V(1 - \mathfrak{f})^n = V_s \tag{6}$$

Therefore,

$$\mathfrak{f} = 1 - \left(\frac{V_s}{V}\right)^{1/n} \tag{7}$$

Equation (7) represents the textbook method for determining the fixed-percentage factor, and the equation is sometimes designated as the *Matheson formula*. A plot showing the change of asset value with time using this declining-balance depreciation method is presented in Fig. 9-1. Comparison with the straight-line method shows that declining-balance depreciation permits the investment to be paid off more rapidly during the early years of life. The increased depreciation costs in the early years are very attractive to concerns just starting in business, because the income-tax load is reduced at the time when it is most necessary to keep all pay-out costs at a minimum.

The textbook relationship presented in Eq. (7) is seldom used in actual practice, because it places too much emphasis on the salvage value of the property and is certainly not applicable if the salvage value is zero. To overcome this disadvantage, the value of the fixed-percentage factor is often chosen arbitrarily using a sound economic basis.

Prior to 1954, the United States government would not accept any depreciation method which permitted depreciation rates more than 50 percent greater than those involved in the straight-line method. In 1954, the laws were changed to allow rates up to twice (200 percent) those for the straight-line method. Under these conditions, one arbitrary method for choosing the value of

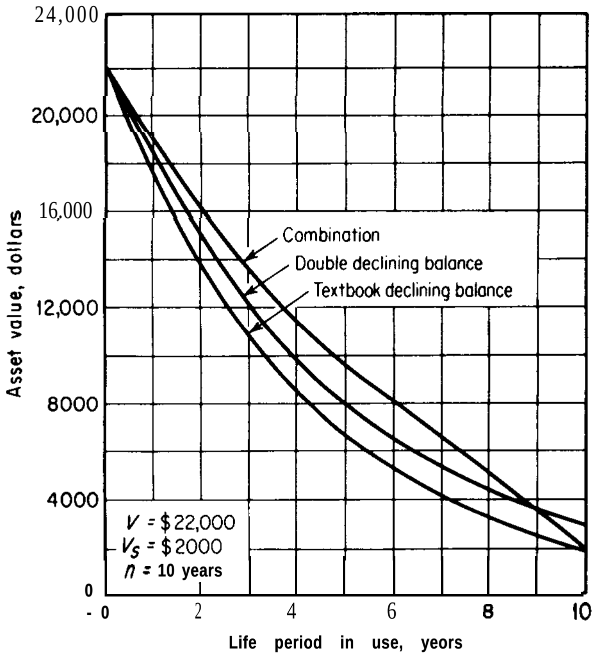

FIGURE 9-2
Types of declining-balance methods for determining depreciation.

f is to fix it at two times the reciprocal of the service life n.† This permits approximately two-thirds of the depreciable value to be written off in the first half of the useful life.‡

Figure 9-2 shows the effect of time on asset value when the declining-balance method of depreciation is used with an arbitrarily chosen value of f. It should be noted that the value of the asset cannot decrease to zero at the end of the service life and may possibly be greater than the salvage or scrap value. To handle this difficulty, it is sometimes desirable to switch from the declining-balance to the straight-line method after a portion of the service life has expired. This is known as the ***combination method.*** It permits the property to be fully depreciated during the service life, yet also gives the advantage of faster early-life write-offs. A curve showing this type of depreciation is presented in Fig. 9-2.

The main advantage of the declining-balance and the combination methods is that they permit greater depreciation allowances in the early life of the

†The salvage value is considered to be zero, and the fixed-percentage factor is based on the straight-line rate of depreciation during the first year.

‡Based on the 1954 tax revision for depreciation accounting, any method could be used if the depreciation for the first two-thirds of the useful life of the property did not exceed the total of such allowances if they had been computed by the double declining-balance method.

property than in the later life. They are particularly applicable for units in which the greater proportion of the production occurs in the early part of the useful life or when operating costs increase markedly with age.

Example 1 Determination of depreciation by straight-line and declining-balance methods. The original value of a piece of equipment is $22,000, completely installed and ready for use. Its salvage value is estimated to be $2000 at the end of a service life estimated to be 10 years. Determine the asset (or book) value of the equipment at the end of 5 years using:

(a) Straight-line method.
(b) Textbook declining-balance method.
(c) Double declining-balance (200 percent) method (i.e., the declining-balance method using a fixed-percentage factor giving a depreciation rate equivalent to twice the minimum rate with the straight-line method).

Solution

(a) Straight-line method:

$$V = \$22{,}000$$

$$V_s = \$2000$$

$$n = 10 \text{ years}$$

$$d = \frac{V - V_s}{n} = \frac{20{,}000}{10} = \$2000/\text{year}$$

Asset value after 5 years = V_a, where a = 5, or

$$V_a = V - ad = 22{,}000 - (5)(2000) = \$12{,}000$$

(b) Textbook declining-balance method:

$$f = 1 - \left(\frac{V_s}{V}\right)^{1/n} = 1 - \left(\frac{2000}{22{,}000}\right)^{1/10} = 0.2131$$

Asset value after 5 years is

$$V_a = V(1 - f)^a = (22{,}000)(1 - 0.2131)^5 = \$6650$$

(c) Double declining-balance (200 percent) method:

Using the straight-line method, the minimum depreciation rate occurs in the first year when V = $22,000 and the depreciation = $2000. This depreciation rate is 2000/22,000, and the double declining-balance (or double fixed-percentage) factor is (2)(2000/22,000) = 0.1818 = f. (It should be noted that the double declining-balance method is often applied to cases where the salvage value is considered to be zero. Under this condition, the double fixed-percentage factor for this example would be 0.2000.)

Asset value after 5 years is

$$V_a = V(1 - f)^a = (22{,}000)(1 - 0.1818)^5 = \$8060$$

Sum-of-the-Years-Digits Method

The sum-of-the-yearsdigits method is an arbitrary process for determining depreciation which gives results similar to those obtained by the declining-balance method. Larger costs for depreciation are allotted during the early-life years than during the later years. This method has the advantage of permitting the asset value to decrease to zero or a given salvage value at the end of the service life.

In the application of the sum-of-the-years-digits method, the annual depreciation is based on the number of service-life years remaining and the sum of the arithmetic series of numbers from 1 to n, where n represents the total service life. The yearly depreciation factor is the number of useful service-life years remaining divided by the sum of the arithmetic series. This factor times the total depreciable value at the start of the service life gives the annual depreciation cost.

As an example, consider the case of a piece of equipment costing $20,000 when new. The service life is estimated to be 5 years and the scrap value $2000. The sum of the arithmetic series of numbers from 1 to n is 1 + 2 + 3 + 4 + 5 = 15. The total depreciable value at the start of the service life is $20,000 − $2000 = $18,000. Therefore, the depreciation cost for the first year is ($18,000)($\frac{5}{15}$) = $6000, and the asset value at the end of the first year is $14,000. The depreciation cost for the second year is ($18,000)($\frac{4}{15}$) = $4800. Similarly, the depreciation costs for the third, fourth, and fifth years, respectively, would be $3600, $2400, and $1200.† Figure 9-1 presents a curve showing the change with time in asset value when the sum-of-the-years-digits method is used for determining depreciation.

Sinking-Fund Method

The use of compound interest is involved in the ***sinking-find method.*** It is assumed that the basic purpose of depreciation allowances is to accumulate a sufficient fund to provide for the recovery of the original capital invested in the property. An ordinary annuity plan is set up wherein a constant amount of money should theoretically be set aside each year. At the end of the service life,

†Equations which apply for determining annual depreciation by the sum-of-the-years-digits method are

$$d_a = \text{depreciation for year } a = \frac{(n-a+1)}{\sum_{1}^{n} a}(V - V_s)$$

$$= \frac{2(n-a+1)}{n(n+1)}(V - V_s)$$

the sum of all the deposits plus accrued interest must equal the total amount of depreciation.

Derivation of the formulas for the sinking-fund method can be accomplished by use of the following notations in addition to those already given:

i = annual interest rate expressed as a fraction
R = uniform annual payments made at end of each year (this is the annual depreciation cost), dollars
$V - V_s$ = total amount of the annuity accumulated in an estimated service life of n years (original value of property minus salvage value at end of service life), dollars

According to the equations developed for an ordinary annuity in Chap. 7 (Interest and Investment Costs),

$$R = (V - V_s)\frac{i}{(1+i)^n - 1} \tag{8}$$

The amount accumulated in the fund after a years of useful life must be equal to the total amount of depreciation up to that time. This is the same as the difference between the original value of the property V at the start of the service life and the asset value V_a at the end of a years. Therefore,

$$\text{Total amount of depreciation after } a \text{ years} = V - V_a \tag{9}$$

$$V - V_a = R\frac{(1+i)^a - 1}{i} \tag{10}$$

Combining Eqs. (8) and (10),

$$V - V_a = (V - V_s)\frac{(1+i)^a - 1}{(1+i)^n - 1} \tag{11}$$

Asset (or book) value after a years = V_a

$$V_a = V - (V - V_s)\frac{(1+i)^a - 1}{(1+i)^n - 1} \tag{12†}$$

Since the value of R represents the annual depreciation cost, the yearly cost for depreciation is constant when the sinking-fund method is used. As shown in Fig. 9-3, this method results in book values which are always greater

†Exactly the same result for asset value after a years is obtained if an annuity due (i.e., equal periodic payments at beginning of each year) is used in place of an ordinary annuity. The periodic payment with an annuity due would be $R/(1+i)$. In accepted engineering practice, the sinking-fund method is based on an ordinary annuity plan.

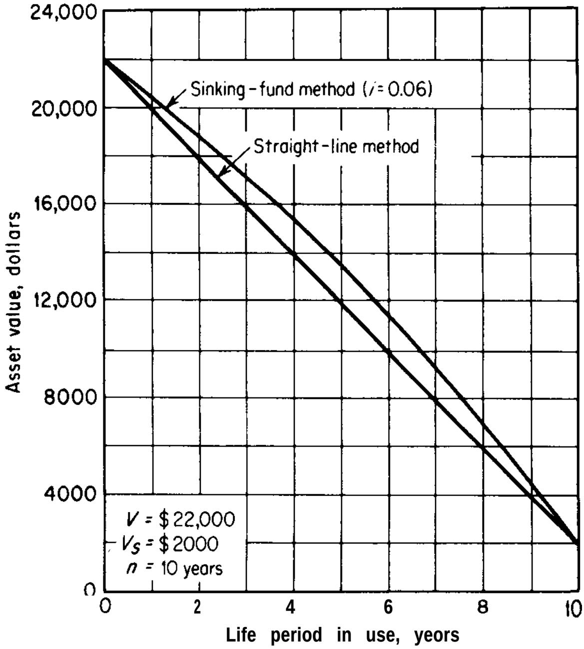

FIGURE 9-3
Asset values of property when depreciated by interest (sinking-fund) and no-interest (straight-line) methods.

than those obtained with the straight-line method. Because of the effects of interest in the sinking-fund method, the annual decrease in asset value of the property is less in the early-life years than in the later years.

Although the sinking-fund viewpoint assumes the existence of a fund into which regular deposits are made, an actual fund is seldom maintained. Instead, the money accumulated from the depreciation charges is put to work in other interests, and the existence of the hypothetical fund merely serves as a basis for this method of depreciation accounting.

The sinking-fund theory of cost accounting is now used by few concerns, although it has seen considerable service in the public-utilities field. Theoretically, the method would be applicable for depreciating any property that did not undergo heavy service demands during its early life and stood little chance of becoming obsolete or losing service value due to other functional causes.

The same approach used in the sinking-fund method may be applied by analyzing depreciation on the basis of reduction with time of future profits obtainable with a property. When this is done, it is necessary to use an interest rate equivalent to the annual rate of return expected from the use of the property. This method is known as the *present-worth method* and gives results similar to those obtained with the conventional sinking-fund approach. The sinking-fund and the present-worth methods are seldom used for depreciation cost accounting but are occasionally applied for purposes of comparing alternative investments.

Accelerated Cost Recovery System

The Accelerated Cost *Recovery* System (ACRS) is a system for determining depreciation allowances based on statutory annual percentages and class life periods established for the United States by Federal income-tax regulations. The basis for the statutory percentage factors is the declining-balance method of depreciation combined with the straight-line method. The original ACRS was in effect by Federal tax laws from 1981 through 1986 with a ***Modified Accelerated Cost Recovery*** System (MACRS) going into effect in 1987.

In general, ACRS allowed one-half of a full year's deduction for property in the year it was placed in service and no deduction in the year when the property was anticipated to be disposed of, although special month-by-month rules could be applied for some cases. Similarly, this so-called "half-year ***convention"*** applied for MACRS as one-half of a full year's deduction for property in the year it was placed in service, but it also allowed one-half of a full year's deduction during the year of disposal. Thus, the years of depreciable values equaled the class years for ACRS and equaled one more than the class years for MACRS.

The bases of calculation for the statutory percentage factors which were applied to the values of the original property to determine the yearly deductions are as follows:

For ACRS, the statutory percentages were based on a HO-percent declining balance with a switch to straight-line depreciation at the time appropriate to

TABLE 3
Statutory percentages for use in the Accelerated Cost Recovery System (ACRS). Property put in service after 1980 and before 1987.

Applicable recovery year	Applicable recovery percentage to give annual depreciation for class life of			
	3 years	5 years	10 years	15 years
1	25	15	8	5
2	38	22	14	10
3	37	21	12	9
4		21	10	8
5		21	10	7
6			10	7
7			9	6
8			9	6
9			9	6
10			9	6
11				6
12				6
13				6
14				6
15				6

maximize the deduction. The half-year convention applied for the first year when property was placed in service. Salvage value was taken as zero.

For MACRS, the statutory percentages were based on a 200-percent declining balance for class lives of 3, 5, 7, and 10 years and a 150-percent declining balance for class lives of 15 and 20 years with a switch to straight-line depreciation at the time appropriate to maximize the deduction. The half-year convention applied for the first year when property was placed in service and also for the year of disposal. Salvage value was taken as zero.

For both ACRS and MACRS, the statutory percentages have been calculated for each of a group of class years, and these, in turn, have been related to values of the *Class Life Accelerated Depreciation Range* (CLADR) as noted earlier. Results are given in Table 3 for ACRS and Table 4 for MACRS.

Details, such as shown in Tables 3 and 4, are presented annually as part of the United States Federal Income Tax Regulations. Tables are also given with conventions other than the half-year conventions for MACRS, such as mid-

TABLE 4
Statutory percentages for use in the Modified Accelerated Cost Recovery System (MACRS). Property put in service after 1986.

Applicable recovery year	Applicable recovery percentage to give annual depreciation for class life of					
	3 years	5 years	7 years	10 years	15 years	20 years
1	33.33	20.00	14.29	10.00	5.00	3.750
2	44.45	32.00	24.49	18.00	9.50	7.219
3	14.81	19.20	17.49	14.40	8.55	6.677
4	7.41	11.52	12.49	11.52	7.70	6.177
5		11.52	8.93	9.22	6.93	5.713
6		5.76	8.92	7.37	6.23	5.285
7			8.93	6.55	5.90	4.888
8			4.46	6.55	5.90	4.522
9				6.56	5.91	4.462
10				6.55	5.90	4.461
11				3.28	5.91	4.462
12					5.90	4.461
13					5.91	4.462
14					5.90	4.461
15					5.91	4.462
16					2.95	4.461
17						4.462
18						4.461
19						4.462
20						4.461
21						2.231

quarter with property placed in service in the first, second, third, or fourth quarter.†

During the period from 1981 through 1986, instead of using the applicable ACRS percentages to determine annual depreciation deductions, corporations were allowed to use straight-line depreciation over the recovery period under the following conditions:

For	Use recovery period of
3-year class-life property	3, 5, or 12 years
5-year class-life property	5, 12, or 25 years
lo-year class-life property	10, 25, or 35 years
15-year class-life property	15, 35, or 45 years

Businesses were expected, in general, to conform with the Modified Accelerated Cost Recovery System to determine depreciation deductions for real and tangible property put into service after 1986 when such deductions were involved in income-tax determinations. Figure 9-4 gives a comparison of depreciation results using methods of ACRS, MACRS, and double declining balance (200 percent) with no salvage value combined with straight line.

Example 2 Determination of percentage factors as given for Modified Accelerated Cost Recovery System. Calculate the percentage factors for a class life of 10 years as presented in Table 4 of this chapter for the Modified Accelerated Cost Recovery System (MACRS). Note that MACRS is based on a 200 percent declining balance for this class life with a switch to straight-line depreciation at the time appropriate to maximize the deduction. It is also based on salvage value being zero. The half-year convention in the first and last years applies. Use an initial property value of $22,000 to permit comparison of results to Fig. 9-4.

Solution. The declining-balance equation to use is (value of property at start of year)$(1 - f)$ = value of property at end of year with f being the declining-balance factor. The 200 percent declining-balance factor is based on two times the minimum depreciation rate which occurs in the first year when V = $22,000 and depreciation is $22,000/10 = $2200. Thus, the 200-percent declining-balance factor is $(2)(2200/22{,}000) = 0.20 = f$ which applies to each full year being considered.

For the first year, with the half-year convention, the investment is considered as being made at the midpoint of the year. Thus, the f which applies for the first year is $(2)(1100/22{,}000) = 0.10$.

Value at start of 1st year or at midpoint of 1st year = $22,000.

Value at end of 1st year = 22,000(1 − 0.10) = $19,800.

Percentage factor for 1st year = [(22,000 − 19,800)/22,000](100) = 10.00%.

†See Prentice-Hall Federal Taxes for the current year, Prentice-Hall Information Services, Paramus, NJ 07652.

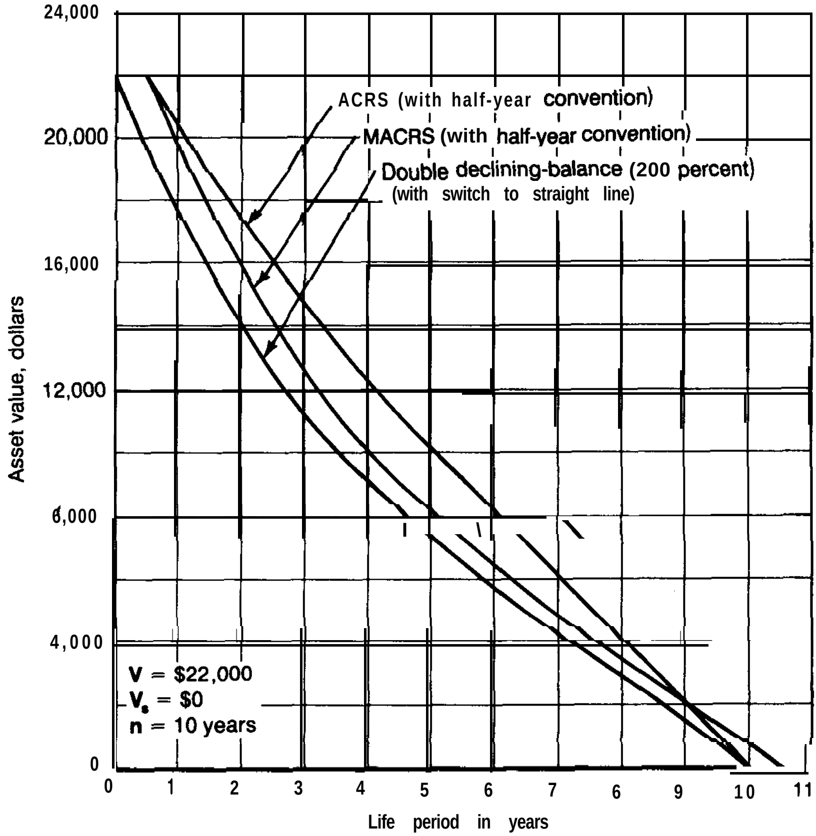

FIGURE 9-4
Asset values of property when depreciated by Accelerated Cost Recovery System (ACRS), Modified Accelerated Cost Recovery System (MACRS), and double declining-balance (200-percent) method with switch to straight-line.

Value at end of 2nd year = 19,800(1 − 0.20) = $15,840.
Percentage factor for 2nd year = [(19,800 − 15,840)/22,000](100) = 18.00%.
Value at end of 3rd year = 15,840(1 − 0.20) = $12,672.
Percentage factor for 3rd year = [(15,840 − 12,672)/22,000](100) = 14.40%.
Value at end of 4th year = 12,672(1 − 0.20) = $10,138.
Percentage factor for 4th year = [(12,672 − 10,138)/22,000](100) = 11.52%.
Value at end of 5th year = 10,138(1 − 0.20) = $8110.
Percentage factor for 5th year = [(10,138 − 8110)/22,000](100) = 9.22%.
Value at end of 6th year = 8110(1 − 0.20) = $6488.
Percentage factor for 6th year = [(8110 − 6488)/22,000](100) = 7.37%.

At this point, any further use of the declining-balance factor for the remaining 4.5 years of property life results in a deduction less than that obtained with the straight-line depreciation method. If one stays with the 200-percent reducing-balance for the 7th year, the amount of depreciation in the 7th year would be 6488 − 6488(1 − 0.20) = $1298 compared to 6488/4.5 = $1442 if the switch is made to straight-line depreciation. Therefore, switch to straight-line method for the remaining 4.5 years of life with the annual depreciation to reduce the property value to zero after 4.5 years being **$1442/year.**

Value at end of 7th year = 6488 − 1442 = $5046.

Percentage factor for 7th year = (1442/22,000)(100) = 6.55% or 6.56%.

Value at end of 8th year = 5046 − 1442 = $3604.

Percentage factor for 8th, 9th, and 10th years = (1442/22,000)(100) = 6.55%.

Value at end of 10th year = 3604 − (2)(1442) = $720.

Percentage factor for 11th year up to half year = (720/22,000)(100) = 6.55/2 = 3.28%.

These percentages agree with those presented in Table 4 of this Chapter, and the year-end values agree with those shown in Fig. 9-4.

SINGLE-UNIT AND GROUP DEPRECIATION

In depreciation accounting procedures, assets may be depreciated on the basis of individual units or on the basis of various types of property groups or classifications. *The single-unit method* requires keeping records on each individual asset. Although the application of this method is simple, the large number of detailed records required makes the accounting expenses very high.

To simplify the accounting procedures, many concerns combine their various assets into groups for depreciation purposes. There are several types of group accounts employed, and the most common among these are composite accounts, classified accounts, and vintage-group accounts.

A *composite* account includes all depreciable assets in one single group, and an overall depreciation rate is applied to the entire account. With this method, the composite depreciation rate must be redetermined when important changes occur in the relative distribution of the service lives of the individual assets.

Instead of including all assets in a single depreciation account, it is possible to classify properties into general types, such as machinery and equipment, office furniture and fixtures, buildings, and transportation equipment. The records for these groups are known as *classified accounts.* A classified account is similar to a composite account because many items are included in the same group, regardless of life characteristics.

Another approach to group depreciation is to include in each account all similar assets having approximately the same service lives. These accounts are known as ***vintage-group accounts.*** A separate record is kept for each group and the same depreciation rate is applied to all the items included in each account. With this method, the advantages of single-unit depreciation are obtained since life characteristics serve as the basis. If a large number of items are contained in a vintage-group account, the overall depreciation results can be quite accurate because the law of averages will apply to the true service lives as compared with the estimated service lives.

ADJUSTMENT OF DEPRECIATION ACCOUNTS

The estimated service life and salvage value of a property are seldom exactly equal to the actual service life and salvage value. It is, therefore, advisable to adjust depreciation accounts by making periodic reestimations of the important variables. When a property is retired under conditions which do not permit exact agreement between estimated and actual values, the difference between the book depreciation and the actual depreciation may be handled in one of the following ways: (1) The gain or loss may be credited or charged on the financial record for the current period; (2) the difference may be credited or charged to a special depreciation reserve; or (3) the difference may be carried on the books for amortization during a reasonable future period.

According to the Federal income-tax laws, any gain on the retirement of a property is taxed as a capital gain. However, losses cannot be subtracted from the taxable income unless the maximum expected life was used. Because of the losses involved when a property must be retired before the end of its estimated service life, some concerns prefer to use a combination of methods 2 and 3 indicated in the preceding paragraph. A special depreciation reserve is built up by continuing the book depreciation of properties whose actual service lives exceed the estimated service lives. This fund is then used to handle losses due to early retirement of assets. The final choice of method for adjusting depreciation accounts depends on the accounting policies of the individual concern and on income-tax regulations.

EVALUATION OF DEPRECIATION METHODS

Comparison of the various depreciation methods shows that the declining-balance and the sum-of-the-years-digits methods give similar results. In both cases, the depreciation costs are greater in the early-life years of the property than in the later years. Annual depreciation costs are constant when the straight-line, sinking-fund, or present-worth method is used. Because interest effects are included in the sinking-fund and present-worth methods, the annual decrease in asset value with these two methods is lower in the early-life years than in the later years. The straight-line method is widely used for depreciation cost accounting because it is very simple to apply, both to groups and single units, and it is acceptable for cost-accounting purposes and for some income-tax determinations.

From the viewpoint of financial protection, it is desirable to make a greater charge for property depreciation during early life than during later life. This can be accomplished by use of the declining-balance or sum-of-the-years-digits method. The difficulties of accurate application to group accounts and income-tax restrictions have served to suppress the widespread usage of these methods. However, in recent years, a large number of industrial concerns have started using declining-balance and sum-of-the-years-digits depreciation, with many companies finding it desirable to use the combination method approved by Federal income-tax regulations of declining balance plus straight-line with statutory percentages for each year based on property life class.

The liberalized tax laws passed in 1954 first permitted use of double declining-balance depreciation as well as sum-of-the-years-digits depreciation for income-tax calculations. In general, these laws gave approval to any depreciation method which did not give faster write-offs during the first two-thirds of an asset's useful life than the double declining-balance method. These regulations were not applicable to assets with service lives of less than 3 years.

The final choice of the best depreciation method depends on a number of different factors. The type and function of the property involved is, of course, one important factor. Also, it is desirable to use a simple formula giving results as accurate as the estimated values involved. The advisability of keeping two separate sets of books, one for income-tax purposes and one for company purposes, should be considered. The final decision involves application of good judgment and an analysis of the existing circumstances.

NOMENCLATURE FOR CHAPTER 9

a = length of time in actual use, years
d = annual depreciation, \$/year
f = fixed-percentage or declining-balance factor
i = annual interest rate expressed as a fraction, percent/100
n = service life, years
R = uniform annual payments made in an ordinary annuity, dollars
V = original value of a property at start of service-life period, completely installed and ready for use, dollars
V_a = asset or book value, dollars
V_s = salvage value of property at end of service life, dollars

PROBLEMS

1. A reactor of special design is the major item of equipment in a small chemical plant. The initial cost of the completely installed reactor is \$60,000, and the salvage value at the end of the useful life is estimated to be \$10,000. Excluding depreciation costs

for the reactor, the total annual expenses for the plant are $100,000. How many years of useful life should be estimated for the reactor if 12 percent of the total annual expenses for the plant are due to the cost for reactor depreciation? The straight-line method for determining depreciation should be used.

2. The initial installed cost for a new piece of equipment is $10,000, and its scrap value at the end of its useful life is estimated to be $2000. The useful life is estimated to be 10 years. After the equipment has been in use for 4 years, it is sold for $7000. The company which originally owned the equipment employs the straight-line method for determining depreciation costs. If the company had used an alternative method for determining depreciation costs, the asset (or book) value for the piece of equipment at the end of 4 years would have been $5240. The total income-tax rate for the company is 34 percent of all gross earnings. Capital-gains taxes amount to 34 percent of the gain. How much net saving after taxes would the company have achieved by using the alternative (in this case, reducing-balance) depreciation method instead of the straight-line depreciation method?

3. A piece of equipment originally costing $40,000 was put into use 12 years ago. At the time the equipment was put into use, the service life was estimated to be 20 years and the salvage and scrap value at the end of the service life were assumed to be zero. On this basis, a straight-line depreciation fund was set up. The equipment can now be sold for $10,000, and a more advanced model can be installed for $55,000. Assuming the depreciation fund is available for use, how much new capital must be supplied to make the purchase?

4. The original investment for an asset was $10,000, and the asset was assumed to have a service life of 12 years with $2000 salvage value at the end of the service life. After the asset has been in use for 5 years, the remaining service life and final salvage value are reestimated at 10 years and $1000, respectively. Under these conditions, what is the depreciation cost during the sixth year of the total life if straight-line depreciation is used?

5. A property has an initial value of $50,000, service life of 20 years, and final salvage value of $4000. It has been proposed to depreciate the property by the text-book declining-balance method. Would this method be acceptable for income-tax purposes if the income-tax laws do not permit annual depreciation rates greater than twice the minimum annual rate with the straight-line method?

6. A piece of equipment having a negligible salvage and scrap value is estimated to have a service life of 10 years. The original cost of the equipment was $40,000. Determine the following:
   (a) The depreciation charge for the fifth year if double declining-balance depreciation is used.
   (*b*) The depreciation charge for the fifth year if sum-of-the-years-digits depreciation is used.
   (*c*) The percent of the original investment paid off in the first half of the service life using the double declining-balance method.
   (*d*) The percent of the original investment paid off in the first half of the service life using the sum-of-the-years-digits method.

7. The original cost of a property is $30,000, and it is depreciated by a 6 percent sinking-fund method. What is the annual depreciation charge if the book value of the property after 10 years is the same as if it had been depreciated at $2500/year by the straight-line method?

8. A concern has a total income of $1 million/year, and all expenses except depreciation amount to $600,000/year. At the start of the first year of the concern's operation, a composite account of all depreciable items shows a value of $850,000, and the overall service life is estimated to be 20 years. The total salvage value at the end of the service life is estimated to be $50,000. Thirty percent of all profits before taxes must be paid out as income taxes. What would be the reduction in income-tax charges for the first year of operation if the sum-of-the-years-digits method were used for depreciation accounting instead of the straight-line method?
9. The total value of a new plant is $2 million. A certificate of necessity has been obtained permitting a write-off of 60 percent of the initial value in 5 years. The balance of the plant requires a write-off period of 15 years. Using the straight-line method and assuming negligible salvage and scrap value, determine the total depreciation cost during the first year.
10. A profit-producing property has an initial value of $50,000, a service life of 10 years, and zero salvage and scrap value. By how much would annual profits before taxes be increased if a 5 percent sinking-fund method *were* used to determine depreciation costs instead of the straight-line method?
11. In order to make it worthwhile to purchase a new piece of equipment, the annual depreciation costs for the equipment cannot exceed $3000 at any time. The original cost of the equipment is $30,000, and it has zero salvage and scrap value. Determine the length of service life necessary if the equipment is depreciated (*a*) by the sum-of-the-years-digits method, and (*b*) by the straight-line method.
12. The owner of a property is using the unit-of-production method for determining depreciation costs. The original value of the property is $55,000. It is estimated that this property can produce 5500 units before its value is reduced to zero: i.e., the depreciation cost per unit produced is $10. The property produces 100 units during the first year, and the production rate is doubled each year for the first 4 years. The production rate obtained in the fourth year is then held constant until the value of the property is paid off. What would have been the annual depreciation cost if the straight-line method based on time had been used?
13. Calculate the percentage factors for a class life of 10 years as presented in Table 3 of this chapter for the Accelerated Cost Recovery System (ACRS). Note that ACRS is based on a HO-percent declining balance with switch to straight-line depreciation at the time appropriate to maximize the deduction. It is also based on salvage value being zero. The half-year convention in the first year applies, but the last half-year deduction cannot be claimed as such. Use an initial property value of $22,000 to permit comparison to Fig. 9-4 and Example 2.
14. A materials-testing machine was purchased for $20,000 and was to be used for 5 years with an expected residual salvage value of $5000. Graph the annual depreciation charges and year-end book values obtained by using:
    (*a*) Straight-line depreciation.
    (*b*) Sum-of-digits depreciation.
    (*c*) Double-declining-balance depreciation.
    (*d*) ACRS with 5-year property recovery.
15. An asset with an original cost of $10,000 and no salvage value has a depreciation charge of $2381 during its second year of *service* when depreciated by the sum-of-digits method. What is its expected useful life?

CHAPTER 10

PROFITABILITY, ALTERNATIVE INVESTMENTS, AND REPLACEMENTS

The word ***profitability*** is used as the general term for the measure of the amount of profit that can be obtained from a given situation. Profitability, therefore, is the common denominator for all business activities.

Before capital is invested in a project or enterprise, it is necessary to know how much profit can be obtained and whether or not it might be more advantageous to invest the capital in another form of enterprise. Thus, the determination and analysis of profits obtainable from the investment of capital and the choice of the best investment among various alternatives are major goals of an economic analysis.

There are many reasons why capital investments are made. Sometimes, the purpose is merely to supply a service which cannot possibly yield a monetary profit, such as the provision of recreation facilities for free use of employees. The profitability for this type of venture cannot be expressed on a positive numerical basis. The design engineer, however, usually deals with investments which are expected to yield a tangible profit.

Because profits and costs are considered which will occur in the future, the possibilities of inflation or deflation affecting future profits and costs must be recognized. The strategy for handling effects of inflation or deflation is discussed in Chap. 11.

Investments may be made for replacing or improving an existing property, for developing a completely new enterprise, or for other purposes wherein a profit is expected from the outlay of capital. For cases of this sort, it is extremely important to make a careful analysis of the capital utilization.

PROFITABILITY STANDARDS

In the process of making an investment decision, the profits anticipated from the investment of funds should be considered in terms of a minimum profitability standard.† This profitability standard, which can normally be expressed on a direct numerical basis, must be weighed against the overall judgment evaluation for the project in making the final decision as to whether or not the project should be undertaken.

The judgment evaluation must be based on the recognition that a quantified profitability standard can serve only as a guide. Thus, it must be recognized that the profit evaluation is based on a prediction of future results so that assumptions are necessarily included. Many intangible factors, such as future changes in demand or prices, possibility of operational failure, or premature obsolescence, cannot be quantitized. It is in areas of this type that judgment becomes critical in making a final investment decision.

A primary factor in the judgment decision is the consideration of possible alternatives. For example, the alternatives to continuing the operation of an existing plant may be to replace it with a more efficient plant, to discontinue the operation entirely, or to make modifications in the existing plant. In reaching the final decision, the alternatives should be considered two at a time on a mutually exclusive basis.

An obvious set of alternatives involves either making the capital investment in a project or investing the capital in a safe venture for which there is essentially no risk and a guaranteed return. In other words, the second alternative involves the company's decision as to the cost of capital.

Cost of Capital

Methods for including the cost of capital in economic analyses have been discussed in Chap. 7. Although the management and stockholders of each company must establish the company's characteristic cost of capital, the simplest approach is to assume that investment of capital is made at a hypothetical cost or rate of return equivalent to the total profit or rate of return over the full expected life of the particular project. This method has the advantage of putting the profitability analysis of all alternative investments on an equal basis, thereby permitting a clear comparison of risk factors. This method is particularly useful for preliminary estimates, but it may need to be refined further to take care of income-tax effects for final evaluation.

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†One often-used basis for the minimum profitability standard is the value of money to the company, expressed as a rate, based on earnings after taxes.

BASES FOR EVALUATING PROJECT PROFITABILITY

Total profit alone cannot be used as the deciding profitability factor in determining if an investment should be made. The profit goal of a company is to maximize income above the cost of the capital which must be invested to generate the income. If the goal were merely to maximize profits, any investment would be accepted which would give a profit, no matter how low the return or how great the cost. For example, suppose that two equally sound investments can be made. One of these requires $100,000 of capital and will yield a profit of $10,000/year, and the second requires $1 million of capital and will yield $25,000/year. The second investment gives a greater yearly profit than the first, but the annual *rate of return* on the second investment is only

$$(\$25{,}000/\$1{,}000{,}000) \times (100) = 2.5 \text{ percent}$$

while the annual rate of return on the $100,000 investment is 10 percent. Because reliable bonds and other conservative investments will yield annual rates of return in the range of 6 to 9 percent, the $1 million investment in this example would not be very attractive; however, the 10 percent return on the $100,000 capital would make this investment worthy of careful consideration. Thus, for this example, the rate of return, rather than the total amount of profit, is the important profitability factor in determining if the investment should be made.

The basic aim of a profitability analysis is to give a measure of the attractiveness of the project for comparison to other possible courses of action. It is, therefore, very important to consider the exact purpose of a profitability analysis before the standard reference or base case is chosen. If the purpose is merely to present the total profitability of a given project, a simple statement of total profit per year or annual rate of return may be satisfactory. On the other hand, if the purpose is to permit comparison of several different projects in which capital might be invested, the method of analysis should be such that all cases are on the same basis so that direct comparison can be made among the appropriate alternatives.

Mathematical Methods for Profitability Evaluation

The most commonly used methods for profitability evaluation, as illustrated in Fig. 10-1, can be categorized under the following headings:

1. Rate of return on investment
2. Discounted cash flow based on full-life performance
3. Net present worth
4. Capitalized costs
5. Payout period

Each of these methods has its advantages and disadvantages, and much has been written on the virtues of the various methods. Because no single method is best for all situations, the engineer should understand the basic ideas involved in each method and be able to choose the one best suited to the needs of the particular situation.

RATE OF RETURN ON INVESTMENT. In engineering economic studies, rate of return on investment is ordinarily expressed on an annual percentage basis. The yearly profit divided by the total initial investment necessary represents the fractional return, and this fraction times 100 is the standard percent *return on investment*.†‡

Profit is defined as the difference between income and expense. Therefore, profit is a function of the quantity of goods or services produced and the selling price. The amount of profit is also affected by the economic efficiency of the operation, and increased profits can be obtained by use of effective methods which reduce operating expenses.

To obtain reliable estimates of investment returns, it is necessary to make accurate predictions of profits and the required investment. To determine the profit, estimates must be made of direct production costs, fixed charges including depreciation, plant overhead costs, and general expenses. Profits may be expressed on a before-tax or after-tax basis, but the conditions should be indicated. Both working capital and fixed capital should be considered in determining the total investment.§

Returns Incorporating Minimum Profits as an Expense

The standard method for reporting rate of return on investment has been outlined in the preceding paragraphs. Another method which is sometimes used for reporting rate of return is based on the assumption that it must be possible to obtain a certain minimum profit or return from an investment before the necessary capital outlay will be desirable. This minimum profit is included as a

†The normal procedure is to base the percent return on investment on the total initial investment. However, because equipment depreciates during its useful life, it is sometimes convenient to base the rate of return on the average estimated investment during the life of the project. With this method, the rate of return is determined by dividing the average annual profit or saving by one-half the initial fixed-capital investment (or initial fixed-capital investment minus the estimated salvage value at the end of the useful life) plus the working-capital investment.

‡An article by J. Linsley, Return on Investment: Discounted and Undiscounted, *Chem. Eng.*, **86**(11):201 (May 21, 1979), suggests that "return on investment" can be defined as net, after-tax profit plus depreciation divided by capital investment. This definition of return on investment where depreciation cash flow is included as part of the return is not used in this book. Instead, this method of handling cash flow is included in the profitability methods reported for discounted-cash-flow Profitability Index and Net Present Worth.

§Under some conditions, such as a profitability analysis based on a small component of an overall operation, the return on investment can be based on the fixed-capital investment instead of the total investment.

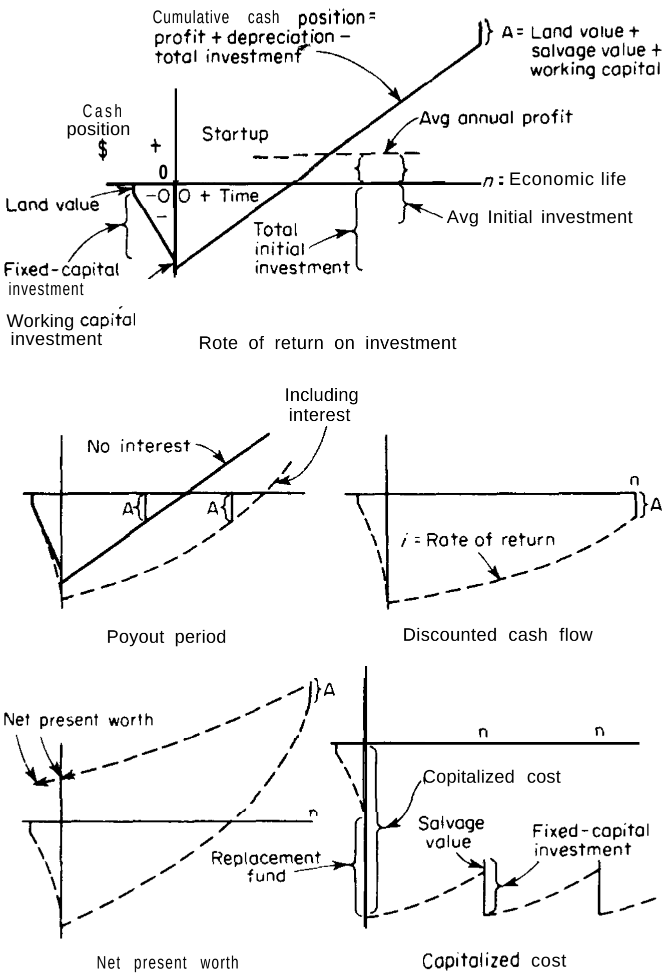

FIGURE 10-1
Graphical description of various profitability measures.

fictitious expense along with the other standard expenses. When return on investment is determined in this manner, the result shows the *risk earning rate*, and represents the return over and above that necessary to make the capital expenditure advisable. If the return is zero or larger, the investment will be attractive. This method is sometimes designated as *return* bused *on capital recovery with minimum profit.*

The inclusion of minimum profit as an expense is rather unrealistic, especially when it is impossible to designate the exact return which would make a given investment worthwhile. One difficulty with this method is the tendency to use a minimum rate of return equal to that obtained from present investments. This, of course, gives no consideration to the element of risk involved in a new venture. Despite these objections, the use of returns incorporating minimum profits as an expense is acceptable providing the base, or minimum return and the general method employed are clearly indicated.

Example 1 Determination of rate of return on investment-consideration of income-tax effects. A proposed manufacturing plant requires an initial fixed-capital investment of $900,000 and $100,000 of working capital. It is estimated that the annual income will be $800,000 and the annual expenses including depreciation will be $520,000 before income taxes. A minimum annual return of 15 percent before income taxes is required before the investment will be worthwhile. Income taxes amount to 34 percent of all pre-tax profits.

Determine the following:

(a) The annual percent return on the total initial investment before income taxes.
(*b*) The annual percent return on the total initial investment after income taxes.
(*c*) The annual percent return on the total initial investment before income taxes based on capital recovery with minimum profit.
(*d*) The annual percent return on the average investment before income taxes assuming straight-line depreciation and zero salvage value.

Solution

(a) Annual profit before income taxes = $800,000 − $520,000 = $280,000.
Annual percent return on the total initial investment before income taxes = [280,000/(900,000 + 100,000)](100) = 28 percent.

(*b*) Annual profit after income taxes = ($280,000)(0.66) = $184,800.
Annual percent return on the total initial investment after income taxes = [184,800/(900,000 + 100,000)](100) = 18.5 percent.

(*c*) Minimum profit required per year before income taxes = ($900,000 + $100,000)(0.15) = $150,000.
Fictitious expenses based on capital recovery with minimum profit = $520,000 + $150,000 = $670,000/year. Annual percent return on the total investment based on capital recovery with minimum annual rate of return of 15 percent before income taxes = [(800,000 − 670,000)/(900,000 + 100,000)](100) = 13 percent.

(*d*) Average investment assuming straight-line depreciation and zero salvage value = $900,000/2 + $100,000 = $550,000.
Annual percent return on average investment before income taxes = (280,000/550,000)(100) = 51 percent.

The methods for determining rate of return, as presented in the preceding sections, give "point values" which are either applicable for one particular year or for some sort of "average" year. They do not consider the time value of money, and they do not account for the fact that profits and costs may vary significantly over the life of the project.

One example of a cost that can vary during the life of a project is depreciation cost. If straight-line depreciation is used, this cost will remain constant; however, it may be advantageous to employ a declining-balance or sum-of-the-years-digits method to determine depreciation costs, which will immediately result in variations in costs and profits from one year to another. Other predictable factors, such as increasing maintenance costs or changing sales volume, may also make it necessary to estimate year-by-year profits with variation during the life of the project. For these situations, analyses of project profitability cannot be made on the basis of one point on a flat time-versus-earning curve, and profitability analyses based on discounted cash flow may be appropriate. Similarly, time-value-of-money considerations may make the discounted-cash-flow approach desirable when annual profits are constant.

DISCOUNTED CASH FLOW

Rate of Return Based on Discounted Cash Flow†

The method of approach for a profitability evaluation by discounted cash flow takes into account the time value of money and is based on the amount of the investment that is unreturned at the end of each year during the estimated life of the project. A trial-and-error procedure is used to establish a rate of return which can be applied to yearly cash flow so that the original investment is reduced to zero (or to salvage and land value plus working-capital investment) during the project life. Thus, the rate of return by this method is equivalent to the maximum interest rate (normally, after taxes) at which money could be borrowed to finance the project under conditions where the net cash flow to the project over its life would be just sufficient to pay all principal and interest accumulated on the outstanding principal.

To illustrate the basic principles involved in discounted-cash-flow calculations and the meaning of rate of return based on discounted cash flow, consider the case of a proposed project for which the following data apply:

Initial fixed-capital investment = \$100,000

Working-capital investment = \$10,000

Service life = 5 years

Salvage value at end of service life = \$10,000

Year	Predicted after-tax cash flow to project based on total income minus all costs except depreciation, \$ (expressed as end-of-year situation)
0	(110,000)
1	30,000
2	31,000
3	36,000
4	40,000
5	43,000

†Common names of methods of return calculations related to the discounted-cash-flow approach are *profitability index, interest rate of return, true rate of return,* and *investor's rate of return.*

TABLE 1
Computation of discounted-cash-flow rate of return

Year (n')	Estimated cash flow to project, $	Trial for $i = 0.15$		Trial for $i = 0.20$		Trial for $i = 0.25$		Trial for $i = 0.207$†	
		Discount factor, $\frac{1}{(1+i)^n}$	Present value, $	Discount factor, $\frac{1}{(1+i)^n}$	Present value, $	Discount factor, $\frac{1}{(1+i)^n}$	Present value, $	Discount factor, $\frac{1}{(1+i)^n}$	Present value, $
0	(110,000)								
1	30,000	0.8696	26.100	0.8333	25,000	0.8000	24,000	0.829	24,900
2	31,000	0.7561	23,400	0.6944	21,500	0.6400	19,800	0.687	21,200
3	36,000	0.6575	23,300	0.5787	20.700	0.5120	18.400	0.570	20,500
4	40,000	0.5718	22,900	0.4623	19.300	0.4096	16,400	0.472	18.800
5	43.000 +20,000	0.4971	31.300	0.4019	25.300	0.3277	20,600	0.391	24,600
	Total		127,000		111.800		99,200		110,000
Ratio = $\frac{\text{total present value}}{\text{initial investment}}$			1.155		1.016		0.902		1.000
								Trial is satisfactory	

†As illustrated in Fig. 10-2, interpolation to determine the correct rate of return can be accomplished by plotting the ratio (total present value/initial investment) versus the trial interest rate for three bracketing values and reading the correct rate from the curve where the ratio = 1.0.

NOTE: In this example, interest was compounded annually on an end-of-year basis and continuous interest compounding was ignored. Also, construction period and land value were not considered. The preceding effects could have been included in the analysis for a more sophisticated treatment using the methods presented in Examples 2 and 3 of this chapter.

Designate the discounted-cash-flow rate of return as i. This rate of return represents the after-tax interest rate at which the investment is repaid by proceeds from the project. It is also the maximum after-tax interest rate at which funds could be borrowed for the investment and just break *even* at the end of the service life.

At the end of five years, the cash flow to the project, compounded on the basis of end-of-year income, will be

$$(\$30,000)(1 + i)^4 + (\$31,000)(1 + i)^3 + (\$36,000)(1 + i)^2 + (\$40,000)(1 + i) + \$43,000 = S \quad (1)$$

The symbol S represents the future worth of the proceeds to the project and must just equal the future worth of the initial investment compounded at an interest rate i corrected for salvage value and working capital. Thus,

$$S = (\$110,000)(1 + i)^5 - \$10,000 - \$10,000 \quad (2)$$

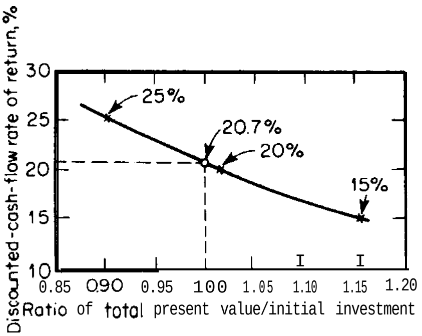

FIGURE 10-2
Graphical analysis for trial-and-error determination of discounted-cash-flow rate of return (see Table 1).

Setting Eq. (1) equal to Eq. (2) and solving by trial and error for i gives $i = 0.207$, or the discounted-cash-flow rate of return is 20.7 percent.

Some of the tedious and time-consuming calculations can be eliminated by applying a ***discount factor*** to the annual cash flows and summing to get a present value equal to the required investment. The discount factor for end-of-year payments and annual compounding is

$$d_{n'} = \frac{1}{(1+i)^{n'}} = \text{discount factor} \tag{3}$$

where i = rate of return

n' = year of project life to which cash flow applies

This discount factor, $d_{n'}$, is the amount that would yield one dollar after n' years if invested at an interest rate of i. The discounted-cash-flow rate of return can be determined by the trial-and-error method illustrated in Table 1, where the annual cash flows are discounted by the appropriate discount factor to a total present value equal to the necessary initial investment.†

Example 2 Discounted-cash-flow calculations based on continuous interest compounding and continuous cash Row. Using the discount factors for continuous interest and continuous cash flow presented in Tables 5 to 8 of Chapter 7, determine the continuous discounted-cash-flow rate of return r for the example presented in the preceding section where yearly cash flow is continuous. The data follow.

Initial tied-capital investment = \$100,000

Working-capital investment = \$10,000

Service life = 5 years

Salvage value at end of service life = \$10,000

†The significance of the use of discount factors, as illustrated in Table 1 and Example 2, can be seen by dividing both sides of Eq. (1) and Eq. (2) by $(1+i)^5$, or by $(1+i)^n$ for the general case where n is the estimated service life in years.

Year	Predicted after-tax cash flow to project based on total income minus all costs except depreciation with cash flow occurring continuously, $ (total of continuous cash flow for year indicated)
1	30,000
2	31,000
3	36,000
4	40,000
5	43,000

Solution. The following tabulation shows the final result of the trial-and-error solution using the factors F_a and F_b from Tables 5 and 6 in Chap. 7:

Year	Estimated continuous cash flow to project, $	Trial for r = 0.225 Discount factor F_b (from Table 6, Chap. 7)	F_a (from Table 5, Chap. 7)	Present value, $
0	(110,000) In an instant			
0-1	30,000	0.8954		26,850
1-2	31,000	0.7151		22,200
2-3	36,000	0.5710		20,550
3-4	40,000	0.4560		18,250
4-5	43,000	0.3648		15,650
5	+20,000 In an instant		0.3246	6,500
				Total 110,000

Trial is satisfactory

Because the assumed trial value of $r = 0.225$ discounted all the cash flows to the present worth of **$110,000,** the continuous interest rate of 22.5 percent represents the discounted-cash-flow rate of return for this example which can be compared to the value of 20.7 percent shown in Table 1 for the case of discrete interest compounding and instantaneous cash flow.

NET PRESENT WORTH

In the preceding treatment of discounted cash flow, the procedure has involved the determination of an index or interest rate which discounts the annual cash flows to a zero present value when properly compared to the initial investment. This index gives the rate of return which includes the profit on the project, payoff of the investment, and normal interest on the investment. A related approach, known as the method of ***net present worth*** (or ***net present value*** or ***venture worth),*** substitutes the cost of capital at an interest rate i for the discounted-cash-flow rate of return. The cost of capital can be taken as the average rate of return the company earns on its capital, or it can be designated as the minimum acceptable return for the project. The net present worth of the project is then the difference between the present value of the annual cash flows and the initial required investment.

To illustrate the method for determining net present worth, consider the example presented in Table 1 for the case where the value of capital to the company is at an interest rate of 15 percent. Under these conditions, the present value of the cash flows is $127,000 and the initial investment is $110,000. Thus, the net present worth of the project is

$127,000 $110,000 = $17,000

Work Sheet for Calculating Present Value and Net Present Worth

An example of a work sheet that can be used for handling discounted-cash-flow presentations to determine present value and net present worth is given in Table 2. The definitions as given in lines 16 and 17 of this table clearly show the preferred distinction between the terms net present *worth* and present value as used in this text. The table is particularly useful because it makes certain the user handles depreciation cash flow correctly by subtracting depreciation costs to determine tax costs (see lines 10 and 11) and including depreciation cash flow to determine the annual cash income (see lines 9 and 12). Line 14 shows four values of discount factors for 15 percent interest based on (*a*) continuous uniform cash flow and continuous interest compounding, (*b*) continuous uniform cash flow and finite (year-end) interest compounding, (*c*) finite (year-end) cash flow and continuous interest compounding, and (*d*) finite (year-end) cash flow and finite (year-end) interest compounding.

Lines 1, 2, and 3 (investments) in Table 2 would normally only be filled in for the first column (discount factor of 1.000) which is designated as the zero year for the operation, with the unit actually going into operation at the start of the so-called first year. It is assumed that working capital and salvage value will be recovered in a lump sum at the end of the estimated service life, so these values are listed on lines 1, 2, and 13 as positive (incoming funds) numbers in the end-of-life column. Since these are lump-sum instantaneous values, the discount factor to apply to them is the finite (year-end) cash flow factor as shown in line 14 in the end-of-life column.

Line 13 gives the annual cash flows for each of the operating years with the zero-year column giving only the total capital investment. In line 16, the present value of the annual cash flows to the project is obtained by summing the individual present values for each year of operation including the present value of the working-capital and salvage-value recovery at the end of the service life. Line 17 merely applies the definition of net present worth as used in this text as the difference between the total present value of the annual cash flows to the project and the initial required investment.

TABLE 2

Work sheet for presenting discounted-cash flow, present-value, and net-present-worth determinations

Project Title: ______________________________

Notes: 1. Dollar values can be in thousands of dollars and rounded to the nearest $1,000.
2. For lines 11 and 14, company policies will dictate which tax rate, interest, and discount factors to use.
3. The estimated service life for this example is taken as 5 years.
4. For lines 5, 6, and 7, see Table 27 of Chapter 6 for estimating information and basis.

Line	Item Numbers in ( ) designate line	Year 1986 0	 1987 1st	 1988 2nd	 1989 3rd	 1990 4th	 1991 5th	End-of-life working capital and salvage value
1.	Fixed-capital investment							
2.	Working capital							
3.	Total capital investment (1 + 2)							
4.	Annual income (sales)							
5.	Annual manufacturing cost							
	(a) Raw materials							
	(b) Labor							
	(c) Utilities							
	(d) Maintenance and repairs							
	(e) Operating supplies							
	(f) Laboratory charges							
	(g) Patents and royalties							
	(h) Local taxes and insurance							
	(i) Plant overhead							
	(j) Other (explain in Notes)							
5-T.	Total of line 5							

6.	Annual general expenses							
	(a) Administrative							
	(b) Distribution and selling							
	(c) Research and development							
	(d) Interest							
	(e) Other (explain in Notes)							
6-T.	Total of line 6							
7.	Total product cost (5-T + 6-T)							
8.	Annual operating income (4 − 7)							
9.	Annual depreciation							
10.	Income before tax (8 − 9)							
11.	Income after 34% tax (0.66 × 10)							
12.	Annual cash income (9 + 11)							
13.	Annual cash flow (3 + 12)							(see heading above)
14.	Discount factors for 15% interest							
	(a) See footnote †	1.000	0.929	0.799	0.688	0.592	0.510	0.472
	(b) See footnote ‡	1.000	0.933	0.812	0.706	0.614	0.534	0.947
	(c) See footnote §	1.000	0.861	0.741	0.638	0.549	0.472	0.472
	(d) See footnote 11	1.000	0.870	0.756	0.658	0.572	0.497	0.497
15.	Annual present value (13 x 14)							

16. TOTAL present value of annual cash flows (sum of line 15 *not* including 0 year)
= ________ in dollars or thousands of dollars

17. Net present worth = total present value of annual cash flows − total capital investment = line 16 − line 3 = ________ in dollars or thousands of dollars

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† Continuous uniform cash flow and continuous nominal interest (r) of 15%.

‡ Continuous uniform cash flow and finite effective interest (i) of 15%.

§ Finite (year-end) cash flow and continuous nominal interest (r) of 15%.

¶ Finite (year-end) cash flow and finite effective interest (i) of 15%.

Notes should be included with the table to explain the basis for special factors used, such as escalation factors, startup costs, and depreciation method. The notes can also be used to explain the methods used for estimating the various items as, for example, note 4 in Table 2 showing the methods used for estimating lines 5, 6, and 7.

The format shown in Table 2 is intended as an example, and a real case would undoubtedly include more columns to represent a life of more than five years. Similarly, capital is normally spent during the period of one or two years before operations begin and sales are made. Thus, the factors in the zero-year column could be changed to values other than 1.000 using methods presented in Chap. 7 (Interest and Investment Costs) as illustrated in Example 3 of this chapter.

CAPITALIZED COSTS†

The ***capitalized-cost*** profitability concept is useful for comparing alternatives which exist as possible investment choices within a single overall project. For example, if a decision based on profitability analysis were to be made as to whether stainless steel or mild steel should be used in a chemical reactor as one part of a chemical plant, capitalized-cost comparison would be a useful and appropriate approach. This particular case is illustrated in Example 9 of Chap. 7.

Capitalized cost related to investment represents the amount of money that must be available initially to purchase the equipment and simultaneously provide sufficient funds for interest accumulation to permit perpetual replacement of the equipment. If only one portion of an overall process to accomplish a set objective is involved and operating costs do not vary, then the alternative giving the least capitalized cost would be the desirable economic choice.

The basic equation for capitalized cost for equipment was developed in Chap. 7 as Eq. (28), which can be written as follows:

$$K = C_V + \frac{C_R}{(1+i)^n - 1} = \frac{C_R(1+i)^n}{(1+i)^n - 1} + V_s \tag{4}$$

where K = capitalized cost
C_V = original cost of equipment
C_R = replacement cost
V_s = salvage value at end of estimated useful life
n = estimated useful life of equipment
i = interest rate

$$\frac{(1+i)^n}{(1+i)^n - 1} = \text{capitalized-cost factor}$$

†For an analysis of the meaning of capitalized costs, development of related equations, and references, see Chap. 7 (Interest and Investment Costs).

Inclusion of Operating Costs in Capitalized-Costs Profitability Evaluation

The capitalized-costs concept can be extended to include operating costs by adding an additional capitalized cost to cover operating costs during the life of the project. Each annual operating cost is considered as equivalent to a necessary piece of equipment that will last one year.†

The procedure is to determine the present or discounted value of each year's cost by the method illustrated in Table 1. The sum of these present values is then capitalized by multiplying by the capitalized-cost factor given with Eq. (4). The total capitalized cost is the sum of the capitalized cost for the initial investment and that for the operating costs plus the working capital. This procedure is illustrated as part of Example 5 in this chapter.

PAYOUT PERIOD

Payout period, or ***payout time***,‡ is defined as the minimum length of time theoretically necessary to recover the original capital investment in the form of cash flow to the project based on total income minus all costs except depreciation. Generally, for this method, original capital investment means only the original, depreciable, fixed-capital investment, and interest effects are neglected. Thus,

$$\begin{matrix}\text{Payout period in years} \\ \text{(no interest charge)}\end{matrix} = \frac{\text{depreciable fixed-capital investment}}{\text{avg profit/yr} + \text{avg depreciation/yr}} \tag{5}$$

Another approach to payout period takes the time value of money into consideration and is designated as ***payout period including interest.*** With this method, an appropriate interest rate is chosen representing the minimum acceptable rate of return. The annual cash flows to the project during the estimated life are discounted at the designated interest rate to permit computation of an average annual figure for profit plus depreciation which reflects the time value of money.§ The time to recover the tied-capital investment plus compounded interest on the total capital investment during the estimated life by

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†If annual operating cost is constant and the cost is considered as an end-of-year cost, the capitalized cost of operation is equal to the annual operating cost divided by i. Continuous interest compounding can be used to resolve the problem of whether an operating cost is an end-of-year or start-of-year cost. The effects of depreciation methods and taxes may be very important when capitalized costs are used to compare design alternatives involving operating costs.

‡Other equivalent names are ***payback period, payback time, payoff period, payoff time,*** and ***cash-recovery period.***

§This discounting procedure is similar to that illustrated in the footnote to part (*a*) of Example 5 in this chapter. Continuous interest tables, such as Tables 5 to 8 in Chap. 7, can also be used.

means of the average annual cash flow is the payout period including interest, or

Payout period including interest

$$= \frac{\begin{matrix}\text{depreciable fixed-} \\ \text{capital investment}\end{matrix} + \begin{matrix}\text{interest on total capital} \\ \text{investment during} \\ \text{estimated service life}\end{matrix}}{(\text{avg profit/yr} + \text{avg depreciation/yr})_{\text{as constant annuity}}} \tag{6}$$

This method tends to increase the payout period above that found with no interest charge and reflects advantages for projects that earn most of their profits during the early years of the service life.

USE OF CONTINUOUS INTEREST COMPOUNDING

In the preceding presentation of methods for profitability evaluation, where interest was considered, it was generally treated as finite-period interest compounded annually. By use of the relationships developed in Chap. 7 (Interest and Investment Costs), it is a simple matter to convert to the case of continuous interest compounding in place of finite interest compounding.

For example, the discount factor $d_{n'} = 1/(1+i)^{n'}$, given as Eq. (3), becomes

$$d_{n'} = \frac{1}{e^{rn'}} \tag{7}$$

for the case of continuous interest compounding with r representing the nominal continuous interest. The preceding equation follows directly from Eq. (18) of Chap. 7.

The application of continuous interest compounding, along with a method of profitability evaluation which includes construction costs and other prestartup costs, is illustrated in the following example.

Example 3 Determination of profitability index with continuous interest compounding and prestartup costs. Determine the discounted-cash-flow rate of return (i.e., the profitability index) for the overall plant project described in the following, and present a plot of cash position versus time to illustrate the solution.

One year prior to startup of the plant, the necessary land is purchased at a cost of $200,000.

During the year prior to the startup, the plant is under construction with money for the construction and related activities flowing out uniformly during the entire year starting at zero dollars and totaling $600,000 for the year.

A working-capital investment of $200,000 is needed at the time the plant starts operation and must be retained indefinitely.

Salvage value for the plant at the end of the estimated useful life is $100,000.

The estimated useful life is 10 years.

Estimations of operating costs, income, and taxes indicate that the annual cash flow to the project (i.e., net profit plus depreciation per year) will be $310,000 flowing uniformly throughout the estimated life. This is an after-tax figure.

The concept of continuous interest compounding and continuous cash flow will be used. Neglect any effects due to inflation or deflation.

Solution. The procedure for this problem is similar to that illustrated in Table 1 in that a trial-and-error method is used with various interest rates until a rate is found which decreases the net cash position to zero at the end of the useful life. Let r represent the profitability index or discounted-cash-flow rate of return with continuous cash flow and continuous interest compounding.

1. Determination of cash position at zero time (i.e., at time of plant startup) in terms of unknown profitability index r.

Land value. The in-an-instant value of the land is \$200,000 one year before the zero reference point of plant startup time. The land value at zero time, therefore, is the future worth of this \$200,000 after one year with continuous interest compounding. Thus, by Eq. (36) of Chap. 7 or part (*e*) of Table 3 in Chap. 7.

$$\text{Compounded land value at zero time} = \$200{,}000(e^r)$$

Construction cost. The total construction cost of the plant during the one year prior to startup is \$600,000 occurring uniformly during the year. The compounded construction cost at zero time, therefore, is the future worth of this \$600,000 after one year flowing uniformly throughout the year with continuous compounding. Thus, by Eq. (37) of Chap. 7 or part (*f*) of Table 3 in Chap. 7.

$$\text{Compounded construction cost at zero time} = \$600{,}000\frac{e^r - 1}{r}$$

Working-capital investment. The working-capital investment of \$200,000 must be supplied at the time of plant startup or at the reference point of zero time.

Summary of cash position at zero time.

Total cash position at zero time

$$= CP_{\text{zero time}} = \$200{,}000(e^r) + \$600{,}000\frac{e^r - 1}{r} + \$200{,}000$$

2. Determination of cash position at end of estimated useful life (i.e., ten years from zero time) in terms of profitability index r. At the end of the useful life with the correct value of r, the total cash position, taking into account the working-capital investment, the salvage value, and the land value, must be zero.

After plant startup, the annual cash flow to the project (i.e., net profit plus depreciation) is \$310,000 flowing continuously and uniformly, and this annual figure is constant throughout the estimated useful life.

The following procedure for evaluating the total cash position at the end of the estimated useful life is analogous to the procedure used in establishing Eqs. (1), (2), (3), and (7) of this chapter.

At the end of each year, the compounded cash flow to the project, with continuous uniform flow and continuous compounding, gives, by Eq. (37) of

Chap. 7 or part (f) of Table 3 in Chap. 7, a future worth $(S_{\text{each year}})$ oft

$$S_{\text{each year}} = \$310{,}000\frac{e^r - 1}{r} \qquad (A)$$

At the end of 10 years, the total future worth (S) of the cash flow to the project, by Eq. (36) of Chap. 7, or part (e) of Table 3 in Chap. 7 becomes

$$S = (\$310{,}000)\frac{e^r - 1}{r}(e^{9r} + e^{8r} + e^{7r} + \cdots + e^r + 1) \qquad (B)$$

The future worth of the total flow to the project after 10 years must be equal to the future worth of the total cash position at zero time $(CP_{\text{zero time}})$ compounded continuously for 10 years minus salvage value, land value, and working-capital investment. Therefore, by Eq. (36) of Chap. 7 or part (e) of Table 3 in Chap. 7,

$$S = (CP_{\text{zero time}})(e^{10r}) - \$100{,}000 - \$200{,}000 - \$200{,}000 \qquad (C)$$

3. Determination of profitability index r. Equating Eq. (B) to Eq. (C) gives the following result with r as the only unknown, and a trial-and-error solution will give the profitability index r.

$$(\$310{,}000)\frac{e^r - 1}{r}(e^{9r} + e^{8r} + e^{7r} + \cdots + e^r + 1) - (CP_{\text{zero time}})(e^{10r})$$
$$+ \$100{,}000 + \$200{,}000 + \$200{,}000 = 0 \qquad (D)$$

The trial-and-error approach can be simplified by dividing Eq. (D) by e^{10r} and substituting the expression for $CP_{\text{zero time}}$ to give the present-value or discounted-cash-flow equation as follows:

$$(\$310{,}000)\frac{e^r - 1}{r}\sum_{n'=1}^{n'=10}\frac{1}{e^{rn'}} - \$200{,}000(e^r) - \$600{,}000\frac{e^r - 1}{r}$$
$$- \$200{,}000 + (\$100{,}000 + \$200{,}000 + \$200{,}000)\frac{1}{e^{10r}} = 0 \qquad (E)$$

where $1/e^{rn'}$ represents the discount factor for continuous cash flow and continuous compounding as given in Eq. (7) of this chapter.

Because the compounded annual flow to the project is constant for each year at $(\$310{,}000)[(e^r - 1)/r]$, the year-by-year use of the discount factor, as illustrated in Table 1 of this chapter, can be replaced by a one-step process wherein the equivalent present value is determined from Eq. (25) of Chap. 7, $P = \bar{R}[(e^{rn} - 1)/re^{rn}]$, with $\bar{R}$ = \$310,000, so that

$$\$310{,}000\frac{e^r - 1}{r}\sum_{n'=1}^{n'=10}\frac{1}{e^{rn'}} = 310{,}000\frac{e^{rn} - 1}{re^{rn}}$$

†The concept of continuous and uniform cash flow with continuous interest compounding is obviously an assumption which is reasonable for some cash flow, such as costs for raw material and labor. However, it is clear that some major portions of the cash flow may not approximate continuous flow. For this reason, the annual cash flow is often estimated as an end-of-year-figure, and the interest factor in Eq. (A) is eliminated.

Similarly, in Eq. (*D*), the future-worth expression for the cash flow, $\$310{,}000[(e^r - 1)/r](e^{9r} + e^{8r} + e^{7r} + \cdots + e^r + 1)$, can be replaced by $\$310{,}000[(e^{rn} - 1)/r]$, as shown by **Eq. (23)** of Chap. 7.

With these simplifications, either Eq. (*D*) or (*E*) can be used for the trial-and-error solution for r. Table 3 shows the method of solution using the present-value Eq. (*E*) to give the correct value of $r = 0.26$.

Thus, the profitability index or discounted-cash-flow rate of return for this example is 26%.

TABLE 3
Computation of profitability index for Example 3†
$n = 10$ years
Basis: Eq. (*E*) and zero (present-value) time at plant startup

Trial for	$r = 0.20$	$r = 0.25$	$r = 0.40$	$r = 0.26$‡
a. Present value of cash flow to project	$\frac{e^r - 1}{r} = 1.107$	1.136	1.230	1.142
$(\$310{,}000)\ \frac{e^r - 1}{r} \sum_{n'=1}^{n'=10} \frac{1}{e^{rn'}}$	$\frac{e^{rn} - 1}{re^{rn}} = 4.31$	3.68	2.45	3.565
or $(\$310{,}000)\ \frac{e^{rn} - 1}{re^{rn}}$	**$1,335,000**	**$1,140,000**	$760,000	**$1,107,000**
b. Present value of land	$e^r = 1.221$	1.284	1.492	1.297
$(\$200{,}000)\ e^r$	$244,000	$257,000	$298,000	$259,000
c. Present value of construction cost	$\frac{e^r - 1}{r} = 1.107$	1.136	1.230	1.142
$(\$600{,}000)\ \frac{e^r - 1}{r}$	$664,000	$682,000	$738,000	$685,000
d. Present value of **working-** capital investment	$200,000	$200,000	$200,000	$200,000
e. Present value of terminal land, working capital, and salvage value based on interest compounded continuously for n years	$\frac{1}{e^{nr}} = 0.135$	0.0822	0.0183	0.744
$(\$500{,}000)\ \frac{1}{e^{nr}}$	$68,000	$41,000	$9,000	$37,000
f. Total of all present values with interest of r. Should be zero at correct value of r. $f = a - b - c - d + e$	$295,000 Rate too low	$42,000 Rate too low	**−$467,000** Rate too high	**$0** Trial is satisfactory

† Graphical methods, special tables for particular cases, MAP1 worksheets and terminology, computer solutions, and rules of thumb are available to simplify the type of calculations illustrated in this table. For example, see G. A. Taylor, "**Managerial** and Engineering Economy: Economic Decision-Making, 2d ed., D. Van Nostrand Company, Inc.. Princeton, New Jersey, 1975.

‡ See Fig. **10-2** for example of graphical interpolation procedure.

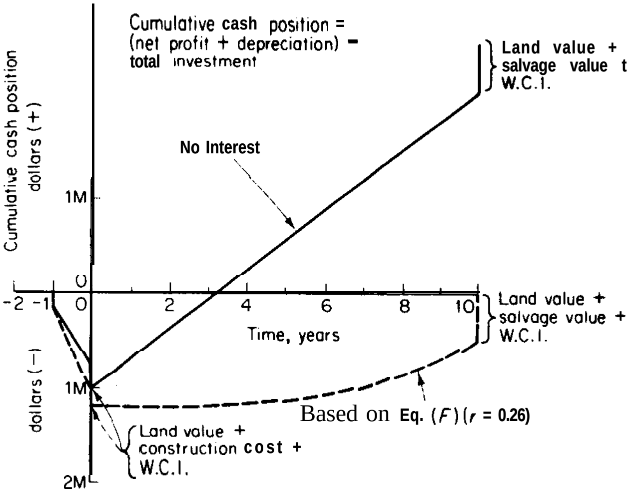

FIGURE 10-3
Illustrative plot showing cash position versus time to explain graphically the solution to Example 3. Dashed line is with interest or profitability index of 26 percent. Solid line is with no interest charge. (Note that method for calculating depreciation is not important except for income taxes.)

4. Graphical representation of problem solution. Equation (D) can be generalized, with the simplifications indicated in the preceding section of this problem, to give Table 3 (note that direct land value, salvage value, and working-capital investment are now included in the cash composition).

Cash position at time n

$$= (\text{annual constant cash flow to project})\frac{e^{rn}-1}{r} - (\text{land value})e^{r(n+Z)} - (\text{construction cost})\frac{e^{rY}-1}{Yr}e^{nr} - (\text{working-capital investment})(e^{nr}) \qquad (F)$$

where Z is the time period in years the land is owned before startup and Y is the time period in years required for construction, In this example, Z and Y are both 1.0.

Figure 10-3 is the requested plot of cash position versus time for the case of $r = 0.26$ based on Eq. (F) and illustrates the concepts involved in the solution of this problem showing the cases with interest (or profitability index) and without interest.

DETERMINING ACCEPTABLE RETURNS

It is often possible to make a profit by the investment of capital, but it is not always easy to determine if a given return is sufficient to justify an investment. Many factors must be considered when deciding if a return is acceptable, and it is not possible to give one figure which will apply for all cases.

When dealing with ordinary industrial operations, profits cannot be predicted with absolute accuracy. Risk factors, therefore, must be given careful consideration, and the degree of uncertainty involved in predicted returns on investments plays an important role in determining what returns are acceptable.

A certain amount of risk is involved in any type of investment, but the degree of risk varies widely for different types of enterprises. For example, there is very little uncertainty in predicting returns on capital invested in government bonds, and the chances of losing the original capital are very small. However, money invested in a wildcat mining enterprise would stand a good chance of being lost completely with no return whatsoever.

If capital is available for investment in a proposed enterprise, it would also be available for use in other ventures. Therefore, a good basis for determining an acceptable return is to compare the predicted return and the risks involved with returns and risks in other types of investments.

Very conservative investments, such as government bonds, pay low returns in the range of 5 to 7 percent, but the risk involved is practically negligible. Preferred stocks yield returns of about 7 to 9 percent. There is some risk involved in preferred-stock investments since a business depression or catastrophe could cause reduction in returns or even a loss of the major portion of the capital investment. Common stocks may yield very high returns; however, the returns fluctuate considerably with varying economic conditions, and there is always the possibility of losing much or all of the original investment.

It can be stated that moderate risks are involved in common-stock investments. Certainly, at least moderate risks are involved in most industrial projects. In general, a 20 percent return before income taxes would be the minimum acceptable return for any type of business proposition, even if the economics appeared to be completely sound and reliable. Many industrial concerns demand a predicted pretax return of at least 30 percent based on reliable economic estimates before they will consider investing capital in projects that are known to be well engineered and well designed.

The final decision as to an acceptable return depends on the probable accuracy of the predicted return and on the amount of risk the investor wishes to take. Availability of capital, personal opinions, and intangible factors, such as the response of the public to changes or appearances, may also have an important effect on the final decision.

ALTERNATIVE INVESTMENTS

In industrial operations, it is often possible to produce equivalent products in different ways. Although the physical results may be approximately the same, the capital required and the expenses involved can vary considerably depending on the particular method chosen. Similarly, alternative methods involving varying capital and expenses can often be used to carry out other types of business ventures. It may be necessary, therefore, not only to decide if a given business venture would be profitable, but also to decide which of several possible methods would be the most desirable.

The final decision as to the best among alternative investments is simplified if it is recognized that each dollar of additional investment should yield an adequate rate of return. In practical situations, there are usually a limited number of choices, and the alternatives must be compared on the basis of incremental increases in the necessary capital investment.

The following simple example illustrates the principle of investment comparison. A chemical company is considering adding a new production unit which will require a total investment of $1,200,000 and will yield an annual profit of $240,000. An alternative addition has been proposed requiring an investment of $2 million and yielding an annual profit of $300,000. Although both of these proposals are based on reliable estimates, the company executives feel that other equally sound investments can be made with at least a 14 percent annual rate of return. Therefore, the minimum rate of return required for the new investment is 14 percent.

The rate of return on the $1,200,000 unit is 20 percent, and that for the alternative addition is 15 percent. Both of these returns exceed the minimum required value, and it might appear that the $2 million investment should be recommended because it yields the greater amount of profit per year. However, a comparison of the incremental investment between the two proposals shows that the extra investment of $800,000 gives a profit of only $60,000, or an incremental return of 7.5 percent. Therefore, if the company has $2 million to invest, it would be more profitable to accept the $1,200,000 proposal and put the other $800,000 in another investment at the indicated 14 percent return.

A general rule for making comparisons of alternative investments can be stated as follows: ***The minimum investment which will give the necessary functional results and the required rate of return should always be accepted unless there is a specific reason for accepting an alternative investment requiring more initial capital.*** When alternatives are available, therefore, the base plan would be that requiring the minimum acceptable investment. The alternatives should be compared with the base plan, and additional capital would not be invested unless an acceptable incremental return or some other distinct advantage could be shown.

Alternatives When an Investment Must Be Made

The design engineer often encounters situations where it is absolutely necessary to make an investment and the only choice available is among various alternatives. An example of this might be found in the design of a plant requiring an evaporation operation. The evaporator units must have a given capacity based on the plant requirements, but there are several alternative methods for carrying out the operation. A single-effect evaporator would be satisfactory. However, the operating expenses would be lower if a multiple-effect evaporator were used, because of the reduction in steam consumption. Under these conditions, the best number of effects could be determined by comparing the increased savings with the investment required for each additional effect. A

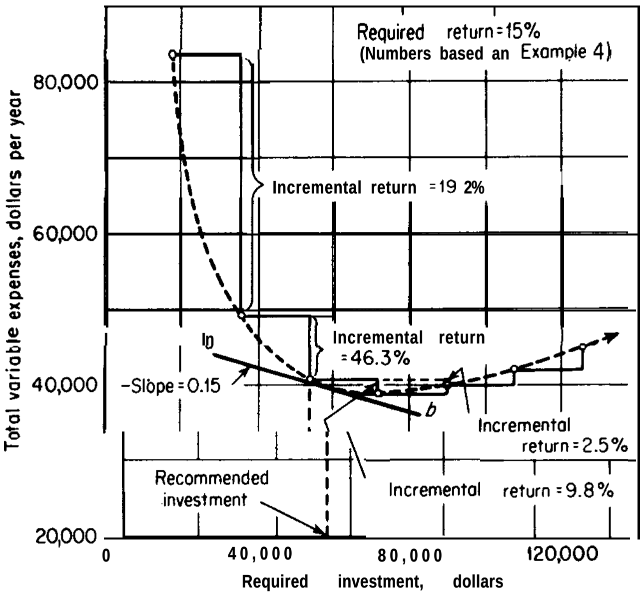

FIGURE 10-4
Comparison of alternative investments when one investment must be made for a given service and there are a limited number of choices.

graphical representation showing this kind of investment comparison is presented in Fig. 10-4.

The base plan for an alternative comparison of the type discussed in the preceding paragraph would be the minimum investment which gives the necessary functional results. The alternatives should then be compared with the base plan, and an additional investment would be recommended only if it would give a definite advantage.

When investment comparisons are made, alternatives requiring more initial capital are compared only with lower investments *which have been found to be acceptable.* Consider an example in which an investment of $50,000 will give a desired result, while alternative investments of $60,000 and $70,000 will give the same result with less annual expense. Suppose that comparison between the $60,000 and the $50,000 cases shows that $60,000 investment to be unacceptable. Certainly, there would be no reason to give further consideration to the $60,000 investment, and the next comparison should be between the $70,000 and the $50,000 cases. This type of reasoning, in which alternatives are compared in pairs on a mutually exclusive basis, is illustrated in the following simplified example.

AN EXAMPLE TO ILLUSTRATE PRINCIPLES OF ALTERNATIVE INVESTMENT ANALYSIS. In making a choice among various alternative investments, it is necessary to recognize the need to compare one investment to another on a mutually exclusive basis in such a manner that the ***return on the incremental investment*** is satisfactory. The following example illustrates this principle.

An existing plant has been operating in such a way that a large amount of heat is being lost in the waste gases. It has been proposed to save money by recovering the heat that is now being lost. Four different heat exchangers have been designed to recover the heat, and all prices, costs, and savings have been calculated for each of the designs. The results of these calculations are presented in the following:

Design	No. 1	No. 2	No. 3	No. 4
Total initial installed cost, $	10,000	16,000	20,000	26,000
Operating costs, $/yr	100	100	100	100
Fixed charges, % of initial cost/yr	20	20	20	20
Value of heat saved, $/yr	4,100	6,000	6,900	8,850

The company in charge of the plant demands at least a 10 percent annual return based on the initial investment for any unnecessary investment. Only one of the four designs can be accepted. Neglecting effects due to income taxes and the time value of money, which (if any) of the four designs should be recommended?

The first step in the solution of this example problem is to determine the amount of money saved per year for each design, from which the annual percent return on the initial investment can be determined. The net annual savings equals the value of heat saved minus the sum of the operating costs and fixed charges; thus,

For design No. 1,

$$\text{Annual savings} = 4100 - (0.2)(10{,}000) - 100 = \$2000$$

$$\text{Annual percent return} = \frac{2000}{10{,}000}(100) = 20\%$$

For design No. 2,

$$\text{Annual savings} = 6000 - (0.2)(16{,}000) - 100 = \$2700$$

$$\text{Annual percent return} = \frac{2700}{16{,}000}(100) = 16.9\%$$

For design No. 3,

$$\text{Annual savings} = 6900 - (0.2)(20{,}000) - 100 = \$2800$$

$$\text{Annual percent return} = \frac{2800}{20{,}000}(100) = 14\%$$

For design No. 4,

$$\text{Annual savings} = 8850 - (0.2)(26{,}000) - 100 = \$3550$$

$$\text{Annual percent return} = \frac{3550}{26{,}000}(100) = 13.6\%$$

Because the indicated percent return for each of the four designs is above the minimum of 10 percent required by the company, any one of the four designs would be acceptable, and it now becomes necessary to choose one of the four alternatives.

ALTERNATIVE ANALYSIS BY METHOD OF RETURN ON INCREMENTAL INVESTMENT. Analysis by means of return on incremental investment is accomplished by a logical step-by-step comparison of an acceptable investment to another which might be better. If design No. 1 is taken as the starting basis, comparison of design No. 2 to design No. 1 shows that the annual saving of \$2700 − \$2000 = \$700 results by making an additional investment of \$16,000 − \$10,000 = \$6,000. Thus, the percent return on the incremental investment is $\frac{700}{6000}(100)$ = 11.7 percent, and design No. 2 is acceptable by company policy in preference to design No. 1. This logical procedure results in the following tabulation and the choice of design No. 2 as the final recommendation:

Design No. 1 is acceptable.

Comparing design No. 1 to design No. 2, annual percent return = $\frac{700}{6000}(100)$ = 11.7 percent. Thus, design No. 2 is acceptable and is preferred over design No. 1.

Comparing design No. 2 to design No. 3, annual percent return = $\frac{100}{4000}(100)$ = 2.5 percent. Thus, design No. 3 compared to design No. 2 shows that the return is unacceptable and design No. 2 is preferred.

Comparing design No. 2 to design No. 4, annual percent return = $\frac{850}{10000}(100)$ = 8.5 percent. Thus, design No. 4 is not acceptable when compared to design No. 2, and design No. 2 is the alternative that should be recommended.

ALTERNATIVE ANALYSIS INCORPORATING MINIMUM RETURN AS A COST. Identical results to the preceding are obtained by choosing the alternative giving the greatest annual profit or saving if the required return is included as a cost for each case. For the heat-exchanger example, the annual cost for the required return would be 10 percent of the total initial investment; thus,

For design No. 1, annual savings above required return = 2000 − (0.1) × (10,000) = \$1000.
For design No. 2, annual savings above required return = 2700 − (0.1) × (16,000) = \$1100.
For design No. 3, annual savings above required return = 2800 − (0.1) × (20,000) = \$800.
For design No. 4, annual savings above required return = 3550 − (0.1) × (26,000) = \$950.

Because annual saving is greatest for design No. 2, this would be the recommended alternative which is the same result as was obtained by the direct analysis based on return on incremental investment.

This simplified example has been used to illustrate the basic concepts involved in making comparisons of alternative investments. The approach was based on using the simple return on initial investment in which time value of money is neglected. Although this method may be satisfactory for preliminary and rough estimations, for final evaluations a more sophisticated approach is needed in which the time value of money is considered along with other practical factors to assure the best possible chance for future success. Typical more advanced approaches of this type are presented in the following sections.

ANALYSIS WITH SMALL INVESTMENT INCREMENTS

The design engineer often encounters the situation in which the addition of small investment increments is possible. For example, in the design of a heat exchanger for recovering waste heat, each square foot of additional heat-transfer area can cause a reduction in the amount of heat lost, but the amount of heat recovered per square foot of heat-transfer area decreases as the area is increased. Since the investment for the heat exchanger is a function of the heat-transfer area, a plot of net savings (or net profit due to the heat exchanger) versus total investment can be made. A smooth curve of the type shown in Fig. 10-5 results.

The point of maximum net savings, as indicated by 0 in Fig. 10-5, represents a classical optimum condition. However, the last incremental investment before this maximum point is attained is at essentially a zero percent return. On the basis of alternative investment comparisons, therefore, some investment less than that for maximum net savings should be recommended.

The exact investment where the incremental return is a given value occurs at the point where the slope of the curve of net savings versus investment equals the required return. Thus, a straight line with a slope equal to the necessary return is tangent to the net-savings-versus-investment curve at the point representing the recommended investment. Such a line for an annual return on incremental investment of 20 percent is shown in Fig. 10-5, and the recommended investment for this case is indicated by *RI*. If an analytical expression

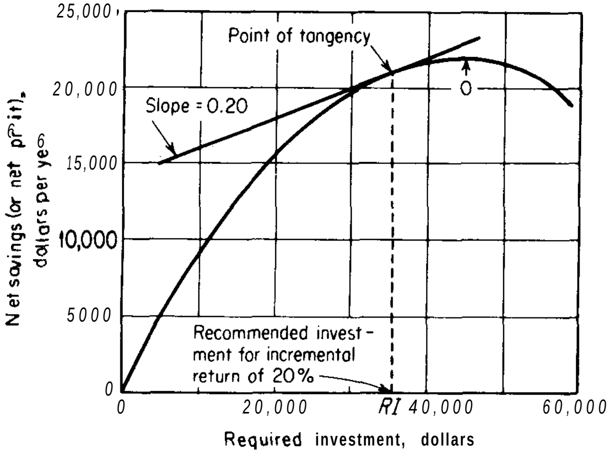

FIGURE 10-5
Graphical method for determining investment for a given incremental return when investment increments can approach zero.

relating net savings and investment is available, it is obvious that the recommended investment can be determined directly by merely setting the derivative of the net savings with respect to the investment equal to the required incremental return.

The method described in the preceding paragraph can also be used for continuous curves of the type represented by the dashed curve in Fig. 10-4. Thus, the line *ab* in Fig. **10-4** is tangent to the dashed curve at the point representing the recommended investment for the case of a 15 percent incremental return.

Example 4 Investment comparison for required operation with limited number of choices. A plant is being designed in which 450,000 lb per 24-h day of a water–caustic soda liquor containing 5 percent by weight caustic soda must be concentrated to 40 percent by weight. A single-effect or multiple-effect evaporator will be used, and a single-effect evaporator of the required capacity requires an initial investment of $18,000. This same investment is required for each additional effect. The service life is estimated to be 10 years, and the salvage value of each effect at the end of the service life is estimated to be $6000. Fixed charges minus depreciation amount to 20 percent yearly, based on the initial investment. Steam costs $0.60 per 1000 lb, and administration, labor, and miscellaneous costs are $40 per day, no matter how many evaporator effects are used.

Where X is the number of evaporator effects, $0.9X$ equals the number of pounds of water evaporated per pound of steam. There are 300 operating days per year. If the minimum acceptable return on any investment is 15 percent, how many effects should be used?

Solution. *Basis:* 1 operating day

X = total number of evaporator effects

$$\text{Depreciation per operating day (straight-line method)} = \frac{X(18{,}000 - 6000)}{(10)(300)}$$

$$= \$4.00X/\text{day}$$

$$\text{Fixed charges} - \text{depreciation} = \frac{X(18{,}000)(0.20)}{300} = \$12.00X/\text{day}$$

$$\text{Pounds of water evaporated per day} = (450{,}000)(0.05)\left(\tfrac{95}{5}\right) - (450{,}000)(0.05)\left(\tfrac{60}{40}\right)$$

$$= 393{,}800 \text{ lb/day}$$

$$\text{Steam costs} = \frac{(393{,}800)(0.60)}{X(0.9)(1000)} = \frac{\$262.5}{X} \text{ per day}$$

X = no. of effects	Steam costs per day	Fixed charges minus depreciation per day	Depreciation per day	Labor, etc., per day	Total cost per day
1	$262.5	$12	$ 4	$40	$318.5
2	131.3	24	8	40	203.3
3	87.5	36	12	40	175.5
4	65.6	48	16	40	169.6
5	52.5	60	20	40	172.5
6	43.8	72	24	40	179.8

Comparing two effects with one effect,

$$\text{Percent return} = \frac{(318.5 - 203.3)(300)(100)}{36{,}000 - 18{,}000} = 192\%$$

Therefore, two effects are better than one effect.

Comparing three effects with two effects,

$$\text{Percent return} = \frac{(203.3 - 175.5)(300)(100)}{54{,}000 - 36{,}000} = 46.3\%$$

Therefore, three effects are better than two effects.

Comparing four effects with three effects,

$$\text{Percent return} = \frac{(175.5 - 169.6)(300)(100)}{72{,}000 - 54{,}000} = 9.8\%$$

Since a return of at least 15 percent is required on any investment, three effects are better than four effects, and the four-effect evaporator should receive no further consideration.

Comparing five effects with three effects,

$$\text{Percent return} = \frac{(175.5 - 172.5)(300)(100)}{90{,}000 - 54{,}000} = 2.5\%$$

Therefore, three effects are better than five effects.

Since the total daily costs continue to increase as the number of effects is increased above five, no further comparisons need to be made.

A three-effect evaporator should be used.

ANALYSIS OF ADVANTAGES AND DISADVANTAGES OF VARIOUS PROFITABILITY MEASURES FOR COMPARING ALTERNATIVES

Of the methods presented for profitability evaluation and the economic comparison of alternatives, net present worth and discounted cash flow are the most generally acceptable, and these methods are recommended. Capitalized costs have limited utility but can serve to give useful and correct results when applied to appropriate situations. Payout period does not adequately consider the later years of the project life, does not consider working capital, and is generally useful only for rough and preliminary analyses. Rates of return on original investment and average investment do not include the time value of money, require approximations for estimating average income, and can give distorted results of methods used for depreciation allowances.

It is quite possible to compare a series of alternative investments by each of the profitability measures outlined in the early part of this chapter and find that different alternatives would be recommended depending on the evaluation technique used.? If there is any question as to which method should be used for a final determination, net ***present worth*** should be chosen, as this will be the most likely to maximize the future worth of the company.

Investment costs due to land can be accounted for in all the methods except payout period. Costs incurred during the construction period prior to startup can be considered correctly in both the net-present-worth and the discounted-cash-flow methods, while they are ignored in the return-on-investment methods and are seldom taken into account in determining payout period. None of the methods gives a direct indication of the magnitude of the project, although net present worth does give a general idea of the magnitude if interpreted correctly. In certain cases, such as for alternatives of different economic lives, the discounted-cash-flow rate-of-return method is very difficult to use for comparing investments correctly. The discounted-cash-flow rate-of-return method may give multiple or impossible answers for unusual cash-flow situations, such as a case where no cash investment is needed at the start and for certain replacement situations.

†This situation is illustrated in Example 5 of this chapter.

TABLE 4
Definitions to clarify income-tax situation for profitability evaluation

Revenue = total income (or total savings)
Net profits = revenue − all expenses − income tax
All expenses = cash expenses + depreciation
Income tax = (revenue − all expenses)(tax rate)
Cash flow = net profits + depreciation
Cash flow = (revenue)(1 − tax rate) − (cash expenses)(1 − tax rate)
+ (depreciation)(tax rate)
Cash flow = (revenue)(1 − tax rate) − (all expenses)(1 − tax rate) + depreciation

For the case of a 34% tax rate

$1.00 of revenue (either as sales income or savings) yields a cash flow of $0.66.
$1.00 of cash expenses (as raw materials, labor, etc.) yields a cash outflow of $0.66.
$1.00 of depreciation yields a cash inflow of $0.34.

Consideration of Income Taxes

Income-tax effects can be included properly in all the profitability methods discussed in this chapter by using appropriate definitions of terms, such as those presented in Table 4. The methods of discounted-cash-flow rate of return and present worth are limited to consideration of cash income and cash outgo over the life of the project. Thus, depreciation, as a cost, does not enter directly into the calculations except as it may affect income taxes.

Net cash flow represents the difference between all cash revenues and all cash expenses with taxes included as a cash expense. Thus, discounted-cash-flow rate of return and present worth should be calculated on an after-tax basis, unless there is some particular reason for a pretax basis, such as comparison to a special alternate which is presented on a pre-tax basis.

Example 5 Comparison of alternative investments by different profitability methods. A company has three alternative investments which are being considered. Because all three investments are for the same type of unit and yield the same service, only one of the investments can be accepted. The risk factors are the same for all three cases. Company policies, based on the current economic situation, dictate that a minimum annual return on the original investment of 15 percent after taxes must be predicted for any unnecessary investment with interest on investment not included as a cost. (This may be assumed to mean that other equally sound investments yielding a 15 percent return after taxes are available.) Company policies also dictate that, where applicable, straight-line depreciation is used and, for time-value of money interpretations, end-of-year cost and profit analysis is used. Land value and prestartup costs can be ignored.

Given the following data, determine which investment, if any, should be made by alternative-analysis profitability-evaluation methods of

(*a*) Rate of return on initial investment
(*b*) Minimum payout period with no interest charge
(*c*) Discounted cash flow
(*d*) Net present worth
(*e*) Capitalized costs

Investment number	Total initial fixed-capital investment, $	Working-capital investment, $	Salvage value at end of service life, $	Service life, years	Annual cash flow to project after taxes,† $	Annual cash expenses‡ (constant for each year), $
1	100,000	10,000	10,000	5	See yearly tabulation §	44,000
2	170,000	10,000	15,000	7	52,000 (constant)	28,000
3	210,000	15,000	20,000	8	59,000 (constant)	21,000

† This is total annual income or revenue minus all costs except depreciation and interest cost for investment.

‡ This is annual coat for operation, maintenance, taxes, and insurance. Equals total annual income minus annual cash flow.

§ For investment number 1, variable annual cash flow to project is: year 1 = $30,000, year 2 = $31,000, year 3 = $36,000, year 4 = $40,000, year 5 = $43,000.

Solution

(a) Method of rate of return on initial investment.

Average annual profit = annual cash flow − annual depreciation cost

The average annual profits for investment No. 1, using straight-line depreciation are as follows:

Year	Average annual profit, dollars	
1	$30{,}000 - \frac{(100{,}000 - 10{,}000)}{5} = 30{,}000 - 18{,}000$	= 12,000
2	31,000 − 18,000	= 13,000
3	36,000 − 18,000	= 18,000
4	40,000 − 18,000	= 22,000
5	43,000 − 18,000	= 25,000
	Total	90,000

For investment No. 1, the arithmetic average of the annual profits is 90,000/5 = $18,000.¶

The annual average rate of return on the first investment is

$$\frac{18{,}000}{100{,}000 + 10{,}000}(100) = 16.4\% \text{ after taxes}$$

¶An alternate method to obtain the average of the annual profits would be to determine the amount of the annuity R based on the end-of-year payments that would compound to the same future worth as the individual profits using an interest rate i of 0.15. With this approach, the average of the annual profits for investment No. 1 would be $17,100.

The method for determining this $17,100 is to apply the series compound-amount factor $[(1+i)^n - 1]/i$ [see Eq. (21) in Chap. 7] to the annuity to give the future worth S of the annual incomes. The expression is $(12{,}000)(1+i)^4 + (13{,}000)(1+i)^3 + (18{,}000)(1+i)^2 + (22{,}000)(1+i) + 25{,}000 = R[(1+i)^5 - 1]/i$. Solving for the case of i = 0.15 gives R = $17,100.

Because this return is greater than 15 percent, one of the three investments will be recommended, and it is only necessary to compare the three investments.

For investment number	Total initial investment	Average annual profit, dollars
1	$110.000	$18,000
2	$180,000	$52,000 - \frac{170,000 - 15,000}{7} = 52,000 - 22,100 = \$29,900$
3	$225,000	$59,000 - \frac{210,000 - 20,000}{8} = 59,000 - 23,800 = 135,200$

Comparing investment No. 2 with investment No. 1,

$$\text{Percent return} = \frac{29,900 - 18,000}{180,000 - 110,000}(100) = 17.0\%$$

Therefore, investment No. 2 is preferred over investment No. 1. Comparing investment No. 3 with investment No. 2,

$$\text{Percent return} = \frac{35,200 - 29,900}{225,000 - 180,000}(100) = 11.8\%$$

This return is not acceptable, and ***investment No. 2 should be recommended.***

The same result would have been obtained if a minimum return of 15 percent had been incorporated as an expense.

(*b*) Method of minimum payout period with no interest charge.

$$\text{Payout period (with no interest charge)} = \frac{\text{depreciable fixed-capital investment}}{\text{avg profit/yr + avg depreciation/yr}}$$

For investment number	***Payout period, years***
1	$\frac{90,000}{18,000 + 18,000} = 2.50$
2	$\frac{155,000}{29,900 + 22,100} = 2.98$
3	$\frac{190,000}{35,200 + 23,800} = 3.22$

The payout period for investment No. 1 is least; therefore, by this method, ***investment No. 1 should be recommended.***

(*c*) Method of discounted cash flow. For investment No. 1, as illustrated in Table 1, the rate of return based on discounted cash flow is 20.7 percent.

For investment No. 2, the discounted-cash-flow equation is

$$(52,000)\left[\frac{1}{1+i} + \frac{1}{(1+i)^2} + \cdots + \frac{1}{(1+i)^7}\right] + (10,000 + 15,000)\frac{1}{(1+i)^7}$$
$$= \$180,000$$

By trial-and-error solution, the discounted-cash-flow rate of return is 22.9 percent

Similarly, for investment No. 3,

$$(59{,}000)\left[\frac{1}{1+i}+\frac{1}{(1+i)^2}+\cdots+\frac{1}{(1+i)^8}\right]+(15{,}000+20{,}000)\frac{1}{(1+i)^8}$$

$$= \$225{,}000$$

By trial-and-error solution, the discounted-cash-flow rate of return is 21.5 percent.

To make a choice among the three alternatives, it is necessary to make a comparison among the three possible choices. This comparison can be made in a relatively straightforward manner using discounted-cash-flow rates of return by comparing pairs of investments on a mutually exclusive basis if the various alternatives have the same economic service lives. When different lengths of service life are involved, as in this problem, the best approach is to avoid the calculated rates of return and make the investment comparison by the net present-worth method as shown in part (*d*) of this problem. It would be possible to use discounted-cash-flow rates of return for comparison between investments with different service lives by assuming that each investment could be repeated at the end of its service life until a common end point was obtained for the investments being compared; however, this method becomes very involved mathematically and is not realistic.

If the service lives of the investments being compared are not widely different, the following ***approximate*** method using discounted-cash-flow rate of return can be employed for the comparison.†

In comparing a pair of alternatives, the base time is chosen as the longer of the two service lines. For the case of the investment with the shorter life, it is assumed that the accumulated amount at the end of its life can be invested at the minimum acceptable rate for the remaining time to equalize the two lives. The rate of return on the incremental investment can then be determined.

Comparison of investment No. 2 to investment No. 1. At the end of its 7-year service life, the net value of investment No. 2 is

$$(180{,}000)(1 + 0.229)’ + 10{,}000 + 15{,}000 = \$785{,}000$$

With investment No. 1, the net value after 7 years is the amount accumulated in 5 years times the factor to let this accumulated amount be invested at 15 percent for 2 more years, or

$$\left[(110{,}000)(1 + 0.207)^5 + 10{,}000 + 10{,}000\right](1 + 0.15)’ = \$398{,}000$$

Therefore, a gain of \$785,000 - \$398,000 = \$387,000 is made in 7 years by an added investment of \$70,000 if investment No. 2 is made instead of investment No, 1. The discounted-cash-flow rate of return for this incremental investment is found by

$$(70{,}000)(1 + i)^7 = 387{,}000$$

$$i = 0.277 \quad \text{or} \quad 27.7\%$$

This return is greater than 15 percent; so investment No. 2 is preferred over investment No. 1.

†The method is shown to illustrate the use of discounted-cash-Row rates of return for investment comparisons. It is correct only for comparisons involving equal service lives. If service lives are different, this method tends to favor the investment with the longest service life.