MODULE 2
SIZE REDUCTION

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SYLLABUS

- Rittinger's law, Kick's law, Bond's law, Work index, Types of comminuting equipment – Jaw Crushers, Gyratory Crusher, Roll crushers; Grinders-hammer Mill, Ball Mill, Rod Mill etc. Dry and wet grinding, open and closed circuit. Simulation of Milling operation grinding rate function, breakage function.

INTRODUCTION

- **Size reduction** is a process in which the particle **size** of a solid is made smaller.
- The term **size reduction** is applied to ways in which particles of solids are cut or broken into smaller pieces.
- **Size reduction** is necessary if the starting material is too coarse, and the final product needs to be a fine powder.
- Comminution, crushing, disintegration, dispersion, grinding and pulverization are the synonyms of size reduction.

ADVANTAGES OF SIZE REDUCTION

- Uniformity in solid masses.
- Uniform flow
- Effective drying
- Surface area increases along with viscosity
- Uniform mixing
- Higher rate of absorption
- Improved dissolution rate.

MECHANISM OF SIZE REDUCTION

- The method of application of the force to the particles may affect the breakage pattern.
- The four basic patterns may be identified, though it is sometimes difficult to identify the dominant mode in any given machine.
- The four basic patterns are:

1. Impact —particle concussion by a single rigid force **(Hammer**).
2. Compression—particle disintegration by two rigid forces **(Nutcracker).**
3. Shear/cutting —produced by a fluid or by particle–particle interaction.
4. Attrition —arising from particles scraping against one another or against a rigid surface **(a File).**

COMPARISON

Impact	Compression	Shear/cutting	Attrition
It gives coarse, medium or fine products.	Used for coarse reduction of hard material to fine size.	Used to obtain definite shape & size product.	Used to obtain fine products of soft, nonabrasive materials

ENERGY REQUIREMENT FOR CRUSHING

- The energy required for reduction in particle size of a solid is directly proportional to the increase in surface area.
- $\frac{dE}{dL} = -CL^p$

p = -1	$E = K_K f_c \ln\frac{L_1}{L_2}$	$C = K_K f_c$		**Kick's law**
p = -2	$E = K_R f_c \left(\frac{1}{L_2} - \frac{1}{L_1}\right)$	$C = K_R f_c$		**Rittinger's law**
p = -3/2	$E = C\left(\frac{1}{L_2^{1/2}} - \frac{1}{L_1^{1/2}}\right)$	$C = \frac{E_i}{\sqrt{10}}$	*Ei the work index*	**Bond's law**

$Where, f_c$ is the crushing strength of the material and
Work index (E_i) **is** the equivalent amount of energy to reduce one ton of the ore from a very large size to 100 um.

ENERGY REQUIREMENT FOR CRUSHING

Bond terms *Ei* the *work index*, and expresses it as the amount of energy required to reduce unit mass of material from an infinite particle size to a size *L*2 of 100 µm.

Laws	Postulation	Equations	Feed size
1. Rittinger's Law (1867)	Grinding work is related to the new surface generated	$W = k_1\left(\frac{1}{x_p} - \frac{1}{x_i}\right)$	Fine grinding (<0.05mm)
2. Kick's law (1885)	Equal work is required to achieve equivalent relative reduction in grain size.	$W = k_2\ ln\frac{x_f}{x_i}$	Course grinding (>50mm)
3. Bond's law (1952)	Grinding work is related to the length of new cracks formed	$W = k_3\left(\frac{1}{\sqrt{x_p}} - \frac{1}{\sqrt{x_i}}\right)$	Intermediate grinding (50mm-0.05mm)

ENERGY UTILISATION

- One of the first important investigations into the distribution of the energy fed into a crusher was carried out by OWENS who concluded that energy was utilized as follows:

a. In producing elastic deformation of the particles before fracture occurs.
b. In producing inelastic deformation which results in size reduction.
c. In causing elastic distortion of the equipment.
d. In friction between particles, and between particles and the machine.
e. In noise, heat and vibration in the plant, and
f. In friction losses in the plant itself.

Note: only about 10 per cent of the total power is usefully employed.

ENERGY UTILISATION

- The new surface produced in size reduction was directly proportional to the energy input. For a given energy input the new surface produced are independent of:

(a) The velocity of impact,

(b) The mass and arrangement of the sample,

(c) The initial particle size, and

(d) The moisture content of the sample.

- The experimental results showed that, provided the new surface did not exceed about 40 m2/kg, the new surface produced was directly proportional to the energy input.

ENERGY REQUIREMENT FOR CRUSHING

Material	Crushing energy	
	Theoretical	Actual
Sodium chloride	0.08 J/m2	90J/m2
Quartz	5J/m2	77J/m2

CRITERIA FOR COMMINUTION

- Crusher and grinders are comminuting equipment.
- These equipment are having very large capacity
- Energy efficient or energy required per unit product size is low.
- The yield obtained are of single size.

CHARACTERISTICS OF COMMINUTED PRODUCT

- Product obtained are not uniform in size. Size analysis is required for such nonuniform product.
- For homogeneous feed, the product is to some extent uniform in size and properties.
- The ratio of largest to smallest particles obtained in comminution is approximately 10^4.

Crushing efficiency

$$\eta_c = \frac{e_s(A_{wp} - A_{wf})}{W_n}$$

e_s= surface energy per unit area

A_{wp}, A_{wf}= area per unit mass of product & feed

W_n= energy absorbed per unit mass

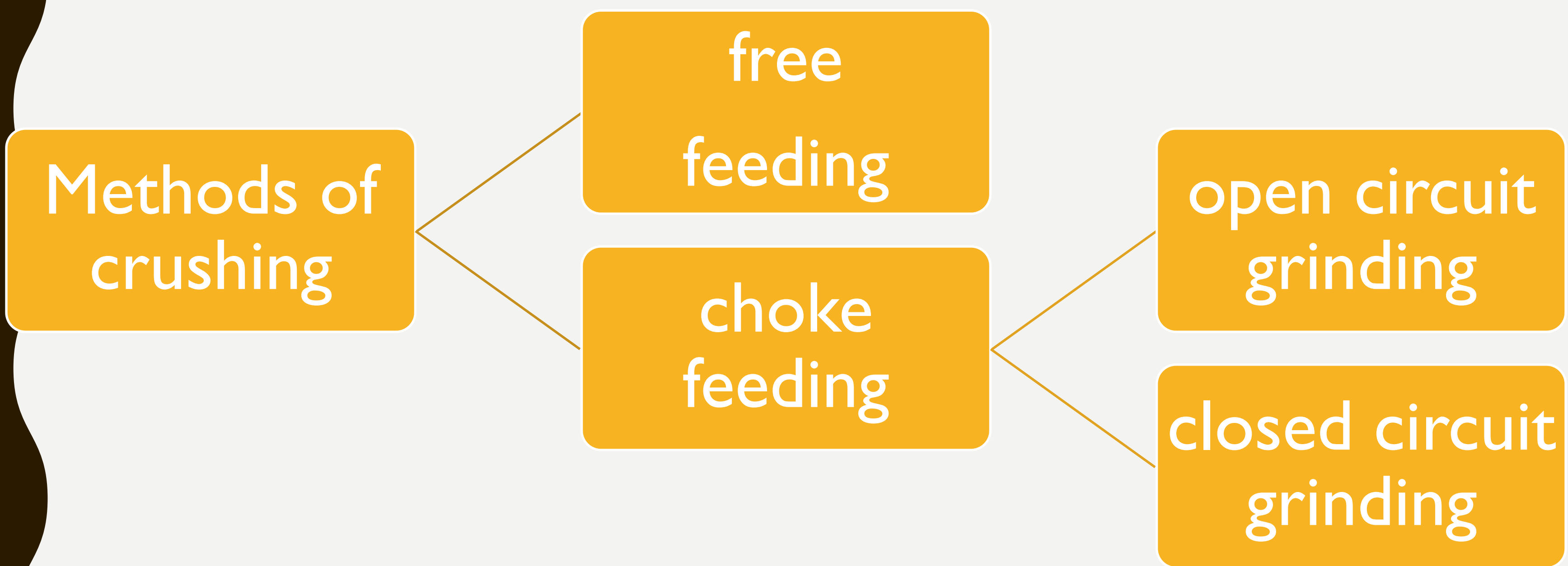
Mechanical efficiency of crushing is

$$\eta_m = \frac{W_n}{W} = \frac{e_s(A_{wp} - A_{wf})}{\eta_c W}$$

$$W = \frac{e_s(A_{wp} - A_{wf})}{\eta_c.\eta_m}$$

W= the energy input required for crushing & to overcome the friction of moving parts of the equipment.

METHODS OF CRUSHING



FREE CRUSHING

- Feeding the material at a comparatively low rate.
- The product can readily escape.
- Its residence time in the machine is therefore short and the production of appreciable quantities of undersize material is avoided.

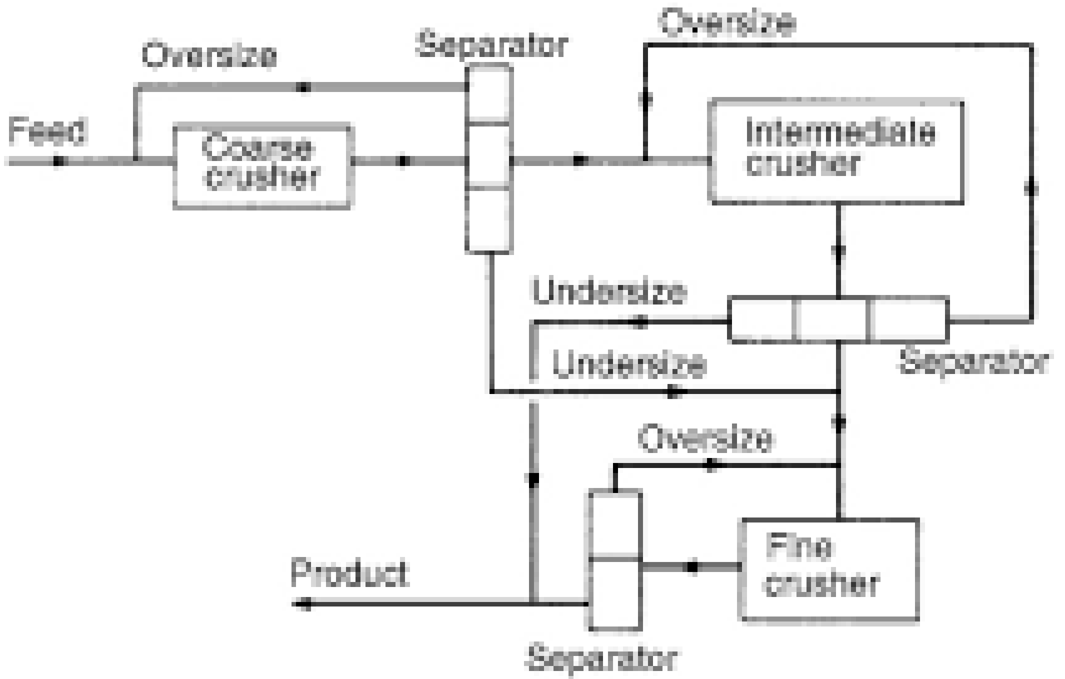
CHOKE FEEDING

- The material remains in the crusher for a longer period.
- The capacity of the machine is low.
- The energy consumption is high because of the cushioning action produced by the accumulated product.
- comparatively small amount of materials is to be crushed.
- This type of feeding is applicable to complete the whole of the size reduction in one operation

TYPES OF CHOKE FEEDING

- **open circuit grinding**: the material is passed only once through the equipment.

  Closed circuit grinding: the product containing material which is insufficiently crushed, the product is separated and return the oversize material for a second crushing.



Separation may be done by:

- allowing the material to fall on to a screen
- or subjecting it to the action of a stream of fluid.

CLASSIFICATION OF SIZE REDUCTION EQUIPMENT

	Feed size	Product size
Coarse crushers	1500–40 mm	50–5 mm
Intermediate crushers	50–5 mm	5–0.1 mm
Fine crushers	5–2 mm	0.1 mm
Colloid mills	0.2 mm	down to 0.01 μm

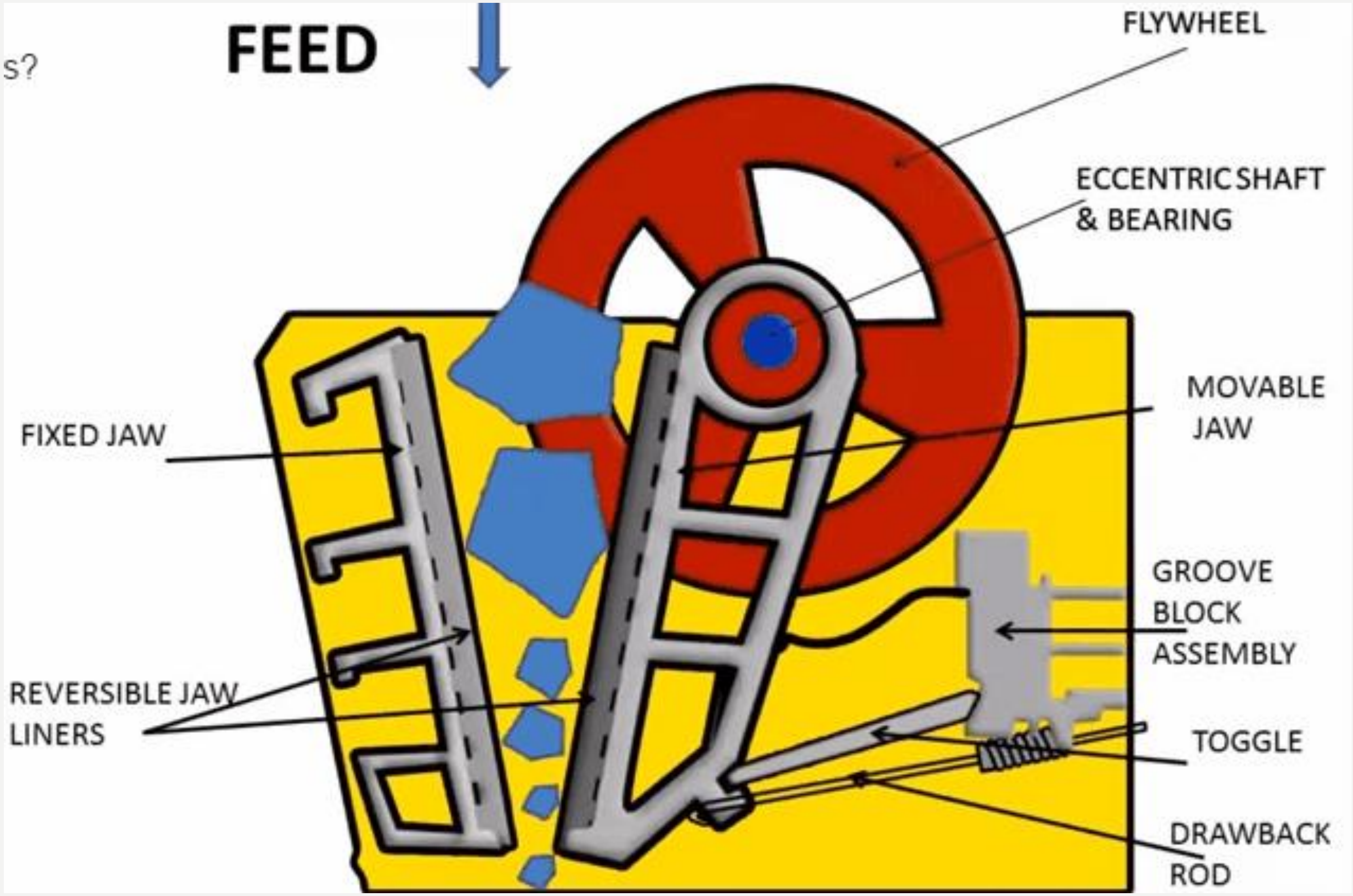
NATURE OF THE MATERIAL TO BE CRUSHED

- *Hardness*
- *Structure*
- *Moisture content*
- *Crushing strength*
- *Friability*
- *Stickiness*
- *Soapiness*
- *Explosive materials*
- *Materials yielding dusts that are harmful to the health*

CRUSHING EQUIPMENT

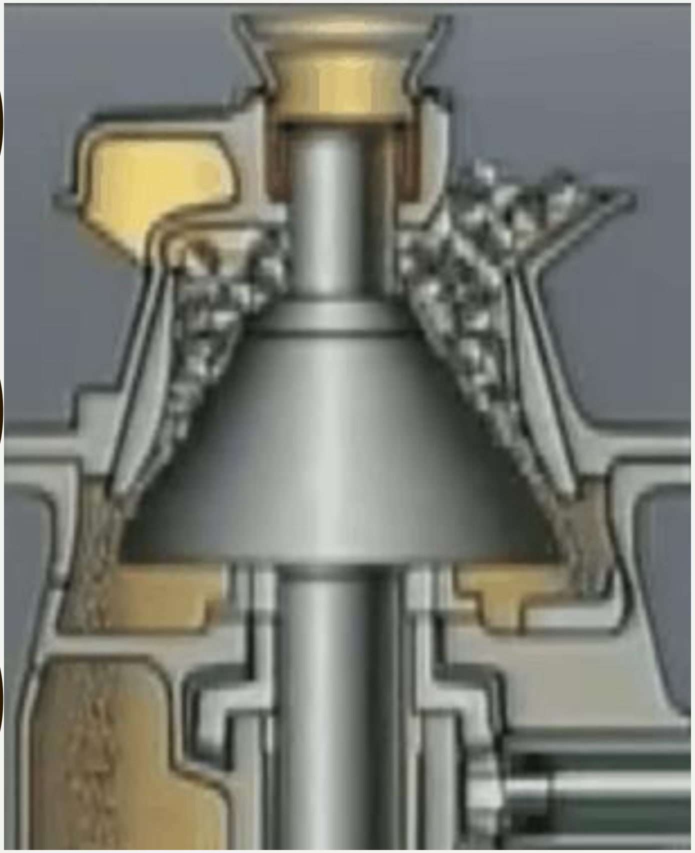
Coarse crushers	Intermediate crushers	Fine crushers
Stag jaw crusher	Crushing rolls	Buhrstone mill
Dodge jaw crusher	Disc crusher	Roller mill
Gyratory crusher	Edge runner mill	NEI pendulum mill
Other coarse crushers	Hammer mill	Griffin mill
	Single roll crusher	Ring roller mill (Lopulco)
	Pin mill	Ball mill
	Symons disc crusher	Tube mill
		Hardinge mill
		Babcock mill

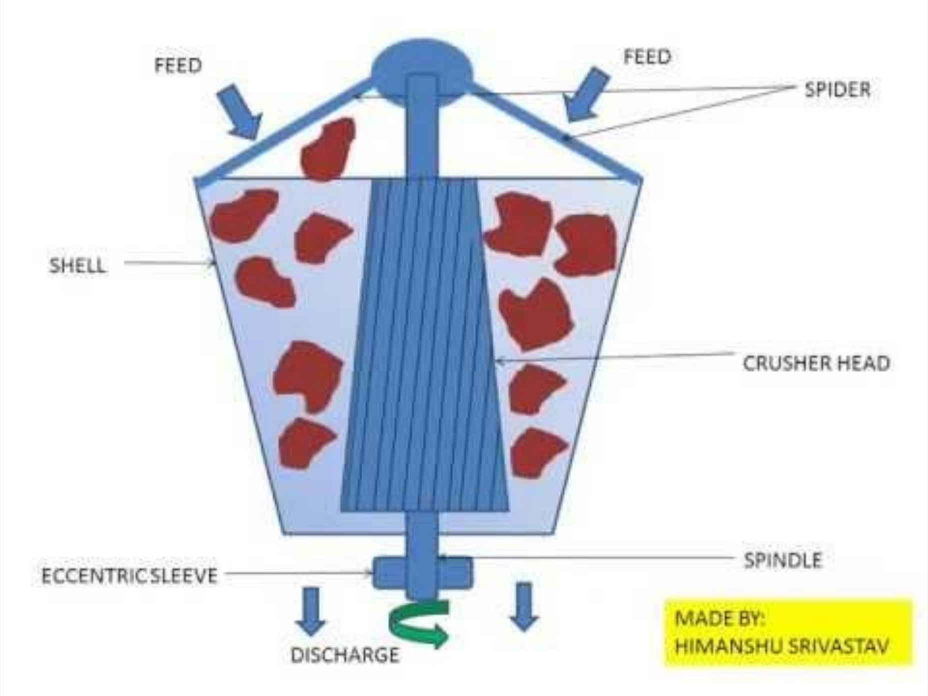
JAW CRUSHER



s?
FEED
FLYWHEEL
ECCENTRIC SHAFT & BEARING
MOVABLE JAW
FIXED JAW
GROOVE BLOCK ASSEMBLY
REVERSIBLE JAW LINERS
TOGGLE
DRAWBACK ROD

GYRATORY CRUSHER



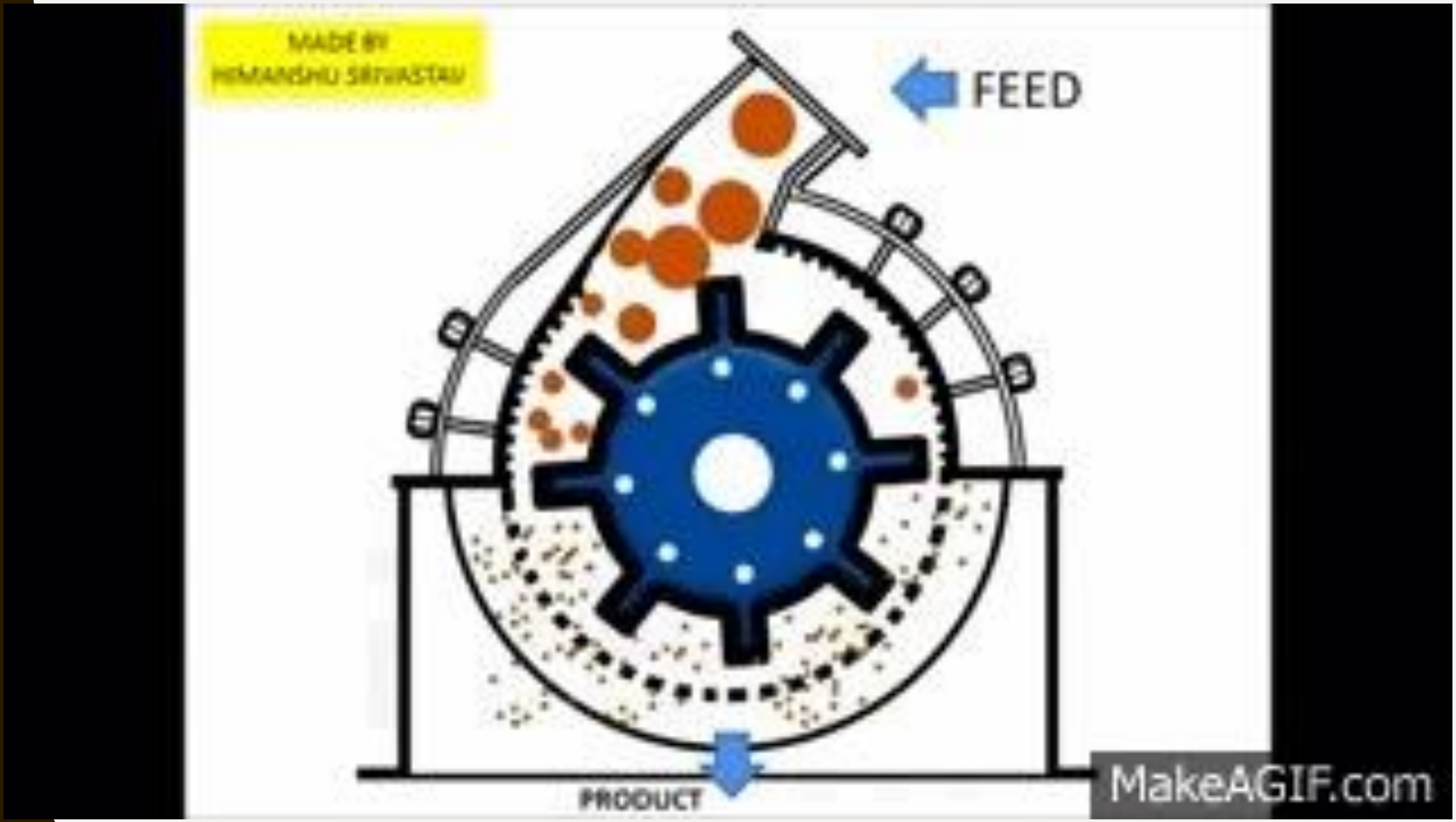


https://www.youtube.com/watch?v=rC9rhEpZBMU

DISCUSS

- Write down the client features of jaw & gyratory crushers?
- What are different types of jaw crusher?
- Discuss the advantages & disadvantages of jaw & gyratory crushers

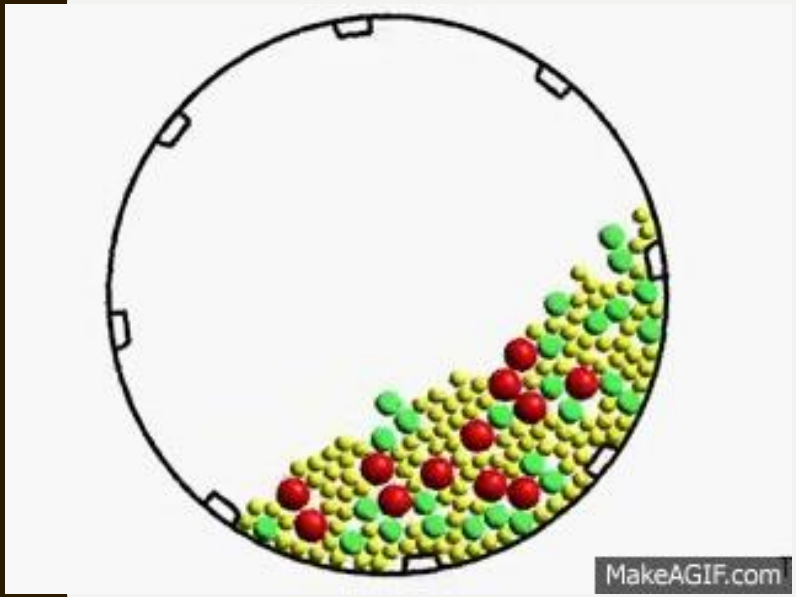
GRINDERS-HAMMER MILL



GRINDERS-HAMMER MILL

- Hammer mills contain a high-speed rotor turning inside a casing.
- The feed dropped from the top & are broken by the sets of swing hammers pinned to a rotor disk.
- During grinding the feed cannot escape and struck by hammers.
- Therefore, particles shatters into pieces and further size is reduced by rubbing action of hammers and pushed through the sieve at discharge opening.
- Generally the diameter of rotor disk is 150-455mm with 4-8 swing hammer. Product obtained are 25mm or 20mesh in size. The speed of hammer tips may reach up to 110m/s. Capacity is 0.1-15 ton/hr for finer sizes then 200mesh (product).
- Tough fibrous solids like leather, soft wet pastes, hard rock can be crushed in hammer mill.
- Commercial mills typically reduce 60 to 240 kg of solid per kWh.

BALL MILL



BALL MILL

- Size reduction occurs by impact of balls in ball mill.
- In a large mill the shell diameter is 3m & length is 4.25m. The balls are of 25-125mm in diameters.
- During the time of rotation, the balls are picked up by the mill and carried nearly to the top, where they break contact with wall and fall to bottom and picked up again.
- Centrifugal force helps the balls to stay connected with wall and with each other during the upward movement.
- Some of the ball do some grinding by slipping & rolling over each other while in contact with wall.
- Increase in speed of mill increases the power consumption.
- Higher speed of ball mill increases the productivity but at very higher speed, the balls are carried over and the situation is called centrifuging.
- The speed at which centrifuging occurs is called critical speed. No grinding is possible at this speed. Therefore, the speed of ball mill must be less than equal to the critical speed.

BALL MILL

- The force balance at A is:
- $mg\ cos\alpha = m\ 4\pi^2 n^2 (R - r)$

Or $cos\alpha = \frac{4\pi^2 n^2 (R-r)}{g}$

At critical speed:

$$\alpha = 0\ \&\ n = n_c = critical\ speed$$

$$n_c = \frac{1}{2\pi}\sqrt{\frac{g}{R - r}}$$

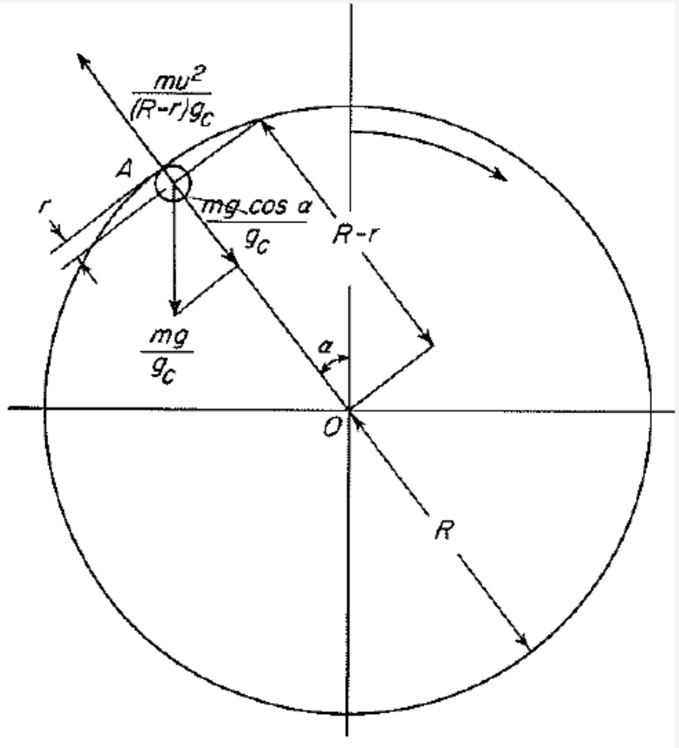


FIGURE 29.10
Forces on ball in ball mill.

ROD/TUBE MILL & BALL MILL

- A tube mill is a continuous mill with a long cylindrical shell.
- The product obtained is 2to 5 times shorter than that obtained from ball mill.
- The working of tube mill is like ball mill.
- The half of the volume of the mill is filled with balls or tubes and void fraction in the mass of balls or tubes at rest is 0.4.
- Both dry & suspension grinding are possible in these mills.
- Rod mills yield 5-200ton/h of 10 mesh product & ball mills produce 1-50tons/h of power of which 70-90% would pass a 200mesh sieve.
- Power required: 4kWh/metric ton for rod miil & 16kWh/metric ton.

OTHER SIZE REDUCTION EQUIPMENT

- Ultrafine grinders: Fluid energy mill, agitated mills, colloid mill (product size 1-20μm)
- Cutting machines : knifes, dicers, slitter etc

DRY & WET GRINDING/MILLING

- For small scale milling the choice between wet & dry milling not makes any significant difference.
- Air is the fluid medium in dry milling and water, or liquid is fluid medium in wet milling.
- In case of dry milling fluid fills the whole equipment. On the other-hand a portion of equipment is filled with liquid in wet grinding.
- Around 30% less energy is consumed in wet grinding than that of dry grinding.

ENERGY CONSUMPTION IN CRASHING

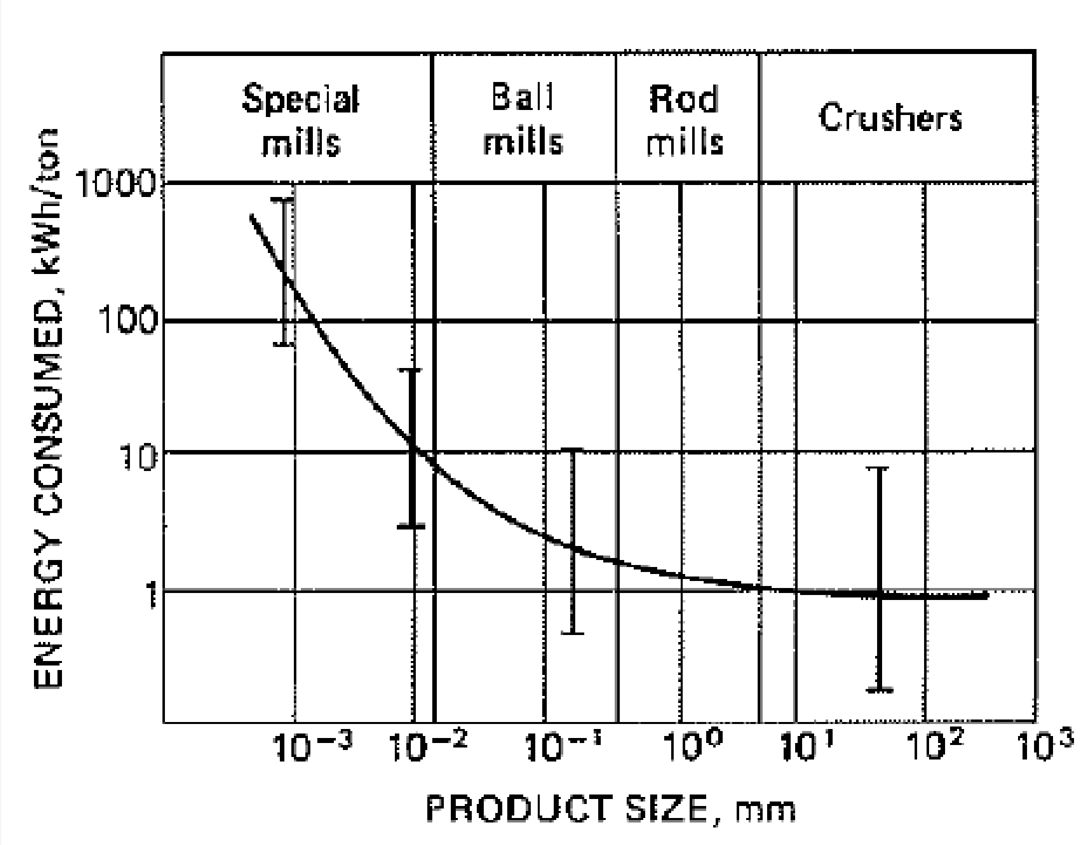


FIGURE 29.16
Energy consumption vs. product size in size-reduction equipment. (*By permission, from Comminution and Energy Consumption, NMAB-264, National Academy Press, 1981.*)

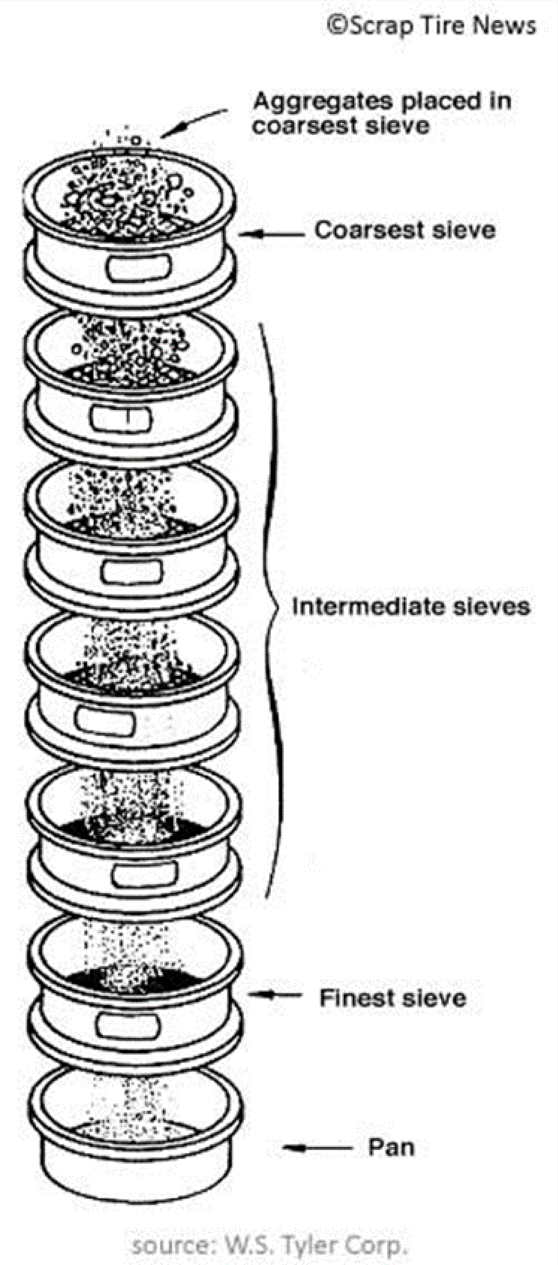
PERFORMANCE OF CRUSHING EQUIPMENT

- For efficiency of crusher depends on:

1. appropriate size & shape of feed.
2. Uniform feed rate
3. Removal of product in regular interval.
4. Separation of unbreakable particles.
5. Crushing with heat removal system for low melting or heat sensitive materials

SIMULATION OF MILLING OPERATION

- The size distribution of products obtained from various size reduction equipment can be predicted by computer simulation.
- The proper mathematical modelling with appropriated assumptions can give actual result.
- In case of crushing, the two controlling parameters are grinding rate function (S_u) & breakage function ($\Delta B_{n,u}$).
- Grinding rate function (S_u) is the fraction of material of given size, coarse than that on screen, which is broken in each time.
- Therefore, $\frac{dx_u}{dt} = -S_u x_u$
- x_u is the mass fraction retained on one of the upper screens.


©Scrap Tire News
Aggregates placed in coarsest sieve
Coarsest sieve
Intermediate sieves
Finest sieve
Pan
source: W.S. Tyler Corp.

SIMULATION OF MILLING OPERATION

- Breakage function ($\Delta B_{n,u}$) gives the size distribution resulting from the breakage of upper material.
- Breakage function depends on size & composition of the materials in the mill.
- There fore, the size distribution equation can be written as:
- $\frac{dx_n}{dt} = -S_u x_n + \sum_{u=1}^{n-1} S_u x_u . \Delta B_{n,u}$
- Assumptions

1. A stack of standard screen is considered. The stack consists of n number of screens. They are numbered from top to bottom i.e n=1 for top or coarsest size.
2. x_u is the mass fraction retained on one of the upper screens ($u < n$).
3. For any fraction except the course, the initial amount is diminished by breakage to smaller sizes and augmented by the creation of new surfaces.
4. For coal, the grinding rate vary with the cube of particle size and breakage function varies with the reduction ratio (Dn/Du).

SIMULATION OF MILLING OPERATION

$B_{n,u} = \left(\frac{D_n}{D_u}\right)^{\beta}$

- $B_{n,u}$ is the total mass fraction smaller than size Dn. It is a cumulative mass fraction or $(\Delta B_{n,u})$.
- β is constant.
- For same reduction ratio $(\Delta B_{n,u})$ will be constant.
- In Eularian method

$$\frac{dx_n}{dt} = \frac{\Delta x_n}{\Delta t} = -S_n x_n + \sum_{u=1}^{n-1} S_u x_u . \Delta B_{n,u}$$

NUMERICAL

A batch grinding mill is charged with material of the composition shown in table. The grinding rate function is 0.001 per sec for 4/6 mesh particles. $\beta = 1.3$ for breakage function. Both breakage function & grinding rate function are independent of time.

(a) How long will it take for the fraction of 4/6 material diminish by 10%?

(b) Tabulate the individual breakage functions for 28/35 mesh fractions and for all coarse fractions.

(c) how will the values of xn vary with time during the first 6h of operation? Use a time interval of 30s in calculation.

Table: initial mass fractions & grinding-rate functions

Mesh	n or u	Dn ot Du (mm)	xn,0	Sn or Su X10^4 (1/sec)
4/6	1	3.327	0.0251	10
6/8	2	2.362	0.1259	3.578
8/10	3	1.651	0.3207	1.222
10/14	4	1.168	0.257	0.4326
14/20	5	0.833	0.159	0.1569
20/28	6	0.589	0.0538	0.0554
28/35	7	0.417	0.021	0.0196

HINTS

(a) $\frac{dx_u}{dt} = -S_u x_u$

$$\therefore t_{req} = -\frac{1}{S_u}\ln\frac{x_{u,final}}{x_{u.initial}} = -\frac{1}{0.001}\ln\frac{0.02259}{0.0251}$$

(b)

(c) $\frac{dx_n}{dt} = \frac{\Delta x_n}{\Delta t} = -S_n x_n + \sum_{u=1}^{n-1} S_u x_u . \Delta B_{n,u}$

$$or\ x_{n,t+1} = x_{n,t} + \Delta t\left(-S_n x_{n,t} + \sum_{u=1}^{n-1} S_u x_{u,t} . \Delta B_{n,u}\right)$$

$$\therefore x_{n,t+1} = (1 - S_n \Delta t) x_{n,t} + \Delta t \sum_{u=1}^{n-1} S_u x_{u,t} . \Delta B_{n,u}$$