CL203 – FLUID MECHANICS

III Semester BTech (Chemical Engineering)

- **Module 4:**
- Flow past of immersed bodies: Introduction;
- concept of drag and lift;
- variation of drag coefficient with Reynolds number;
- streamlining;
- packed bed; concept of equivalent diameter and sphericity;
- Ergun equation,
- Fluidization: Introduction; different types of fluidization;
- fluidized bed assembly; governing equation; industrial use.
- Agitation and mixing of liquids: agitated vessel, blending & mixing, suspension of solid particles.
- Dispersion operation. Turbine Design/scale up, Flow number, Power Requirement. [8]

LECTURE PLAN AND LEARNING OBJECTIVES FOR 40 [ONE HOUR] LECTURES

Module 4: Flow past of immersed bodies: Introduction; concept of drag and lift; variation of drag coefficient with Reynolds number; streamlining; packed bed; concept of equivalent diameter and sphericity; Ergun equation, Fluidization: Introduction; different types of fluidization; fluidized bed assembly; governing equation; industrial use. Agitation and mixing of liquids: agitated vessel, blending & mixing, suspension of solid particles. Dispersion operation. Turbine Design/scale up, Flow number, Power Requirement. [8]

Lecture I

Flow past of immersed bodies: Introduction; concept of drag and lift.

Lecture II

Variation of drag coefficient with Reynolds number; streamlining; packed bed.

Lecture III

Concept of equivalent diameter and sphericity; Ergun equation.

Lecture IV

Fluidization: Introduction; different types of fluidization.

Lecture V

Fluidized bed assembly; governing equation; industrial use.

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Lecture VI

Agitation and mixing of liquids: agitated vessel, blending & mixing, suspension of solid particles. Dispersion operation.

Lecture VII

Agitation and mixing of liquids: agitated vessel, blending & mixing, suspension of solid particles. Dispersion operation.

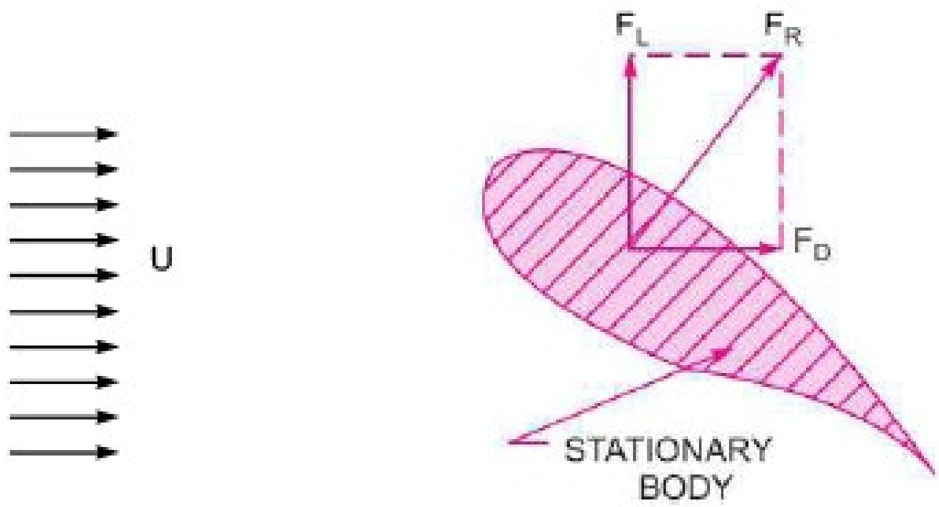
Lecture VIII

Turbine Design/scale up, Flow number, Power Requirement.

- **Flow past immersed bodies : Introduction**
- In this module we will consider various aspects of the flow over bodies that are immersed in a fluid.
- In these situations the object is completely surrounded by the fluid, and the flows are termed as External flows.
- When a fluid is flowing over a stationary body, a force is exerted by the fluid on the body.
- Similarly, when a body is moving in a stationary fluid, a force is exerted by the fluid on the body.
- Also when the body and fluid both are moving at different velocities, a force is exerted by the fluid on the body.
- Some of the examples of the fluids flowing over stationary bodies or bodies moving in a stationary fluids are:
- 1. Flow of air over buildings
- 2. Flow of water over bridges
- 3. Submarines, ships, airplanes moving through water or air.

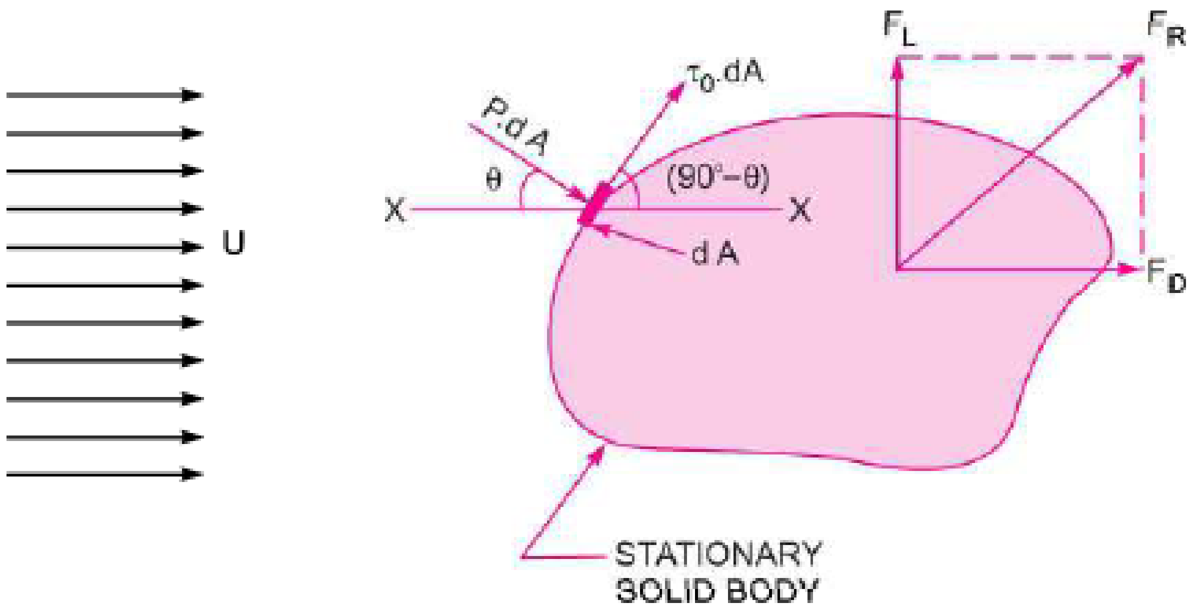
- **Flow past immersed bodies : Introduction**
- The magnitude of this force depends on many factors—certainly the relative velocity V, but also the body shape and size, and the fluid properties (ρ, μ , etc.).
- As the fluid flows around the body, it will generate surface stresses on each element of the surface.
- The surface stresses are composed of tangential stresses due to viscous action and normal stresses due to the local pressure.
- Theoretical (i.e., analytical and numerical) techniques can provide much of the needed information about such flows.
- However, because of the complexities of the governing equations and the complexities of the geometry of the objects involved, the amount of information obtained from purely theoretical methods is limited.
- For these reasons we must resort to experimental methods to determine the net force for most body shapes.
- Traditionally the net force F is resolved into the **drag force, F_D**, defined as the component of the force parallel to the direction of motion, and the **lift force, F_L** (if it exists for a body), defined as the component of the force perpendicular to the direction of motion.

- **Flow past immersed bodies : Introduction**
- **<u>Force exerted by a flowing fluid over a stationary body:</u>**
- Consider a body held stationary in a real fluid, which is flowing at a uniform velocity U.
- The fluid will exert a force on the stationary body.
- The total force F_R exerted by the fluid on the body is perpendicular to the surface of the body.
- Thus the total force is inclined to the direction of motion.
- The total force can be resolved in two components, one in the direction of motion and the other perpendicular to the direction of motion.



- **Flow past immersed bodies : Introduction**
- **Force exerted by a flowing fluid over a stationary body:**
- **DRAG**
- The component of the total force in the direction of motion is called DRAG.
- Thus drag is the force exerted by the fluid in the direction of motion.
- **LIFT:**
- The component of total force in the direction perpendicular to the direction of motion is known as LIFT.
- Thus lift is the force exerted by the fluid in the direction perpendicular to the direction of motion.
- Lift force occurs only when the axis of the body is inclined to the direction of fluid flow.
- If the axis of the body is parallel to the direction of fluid flow, lift force is zero. In that case only drag force acts.

- **Flow past immersed bodies : Introduction**
- **<u>Expression for DRAG and LIFT:</u>**
- Consider an arbitrary shaped solid body placed in a real fluid, which is flowing with a uniform velocity U in a horizontal direction.
- Consider a small elemental area dA on the surface on the body.
- The forces acting on the surface area dA are:
- 1. Pressure force equal to p x dA, acting perpendicular to the surface
- 2. Shear force equal to τ_0 x dA, acting along the tangential direction to the surface.
- Let θ be the angle made by pressure force with horizontal direction.



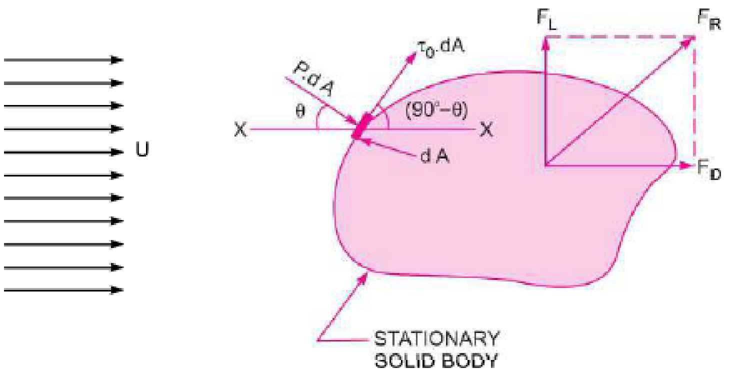
- **Flow past immersed bodies : Introduction**
- **<u>Expression for DRAG and LIFT:</u>**
- (a) Drag Force (F_D):
- The drag force on elemental area = Force due to pressure in the direction of fluid motion + Force due to shear stress in the direction of fluid motion.

$$= pdA\cos\theta + \tau_0 dA\cos(90^\circ - \theta) = pdA\cos\theta + \tau_0 dA\sin\theta$$

Total drag, F_D = Summation of $pdA\cos\theta$ + Summation of $\tau_0 dA\sin\theta$

$$= \int p\cos\theta dA + \int \tau_0 \sin\theta dA.$$

The term $\int p\cos\theta dA$ is called the pressure drag or form drag while the term $\int \tau_0 \sin\theta dA$ is called the friction drag or skin drag or shear drag.



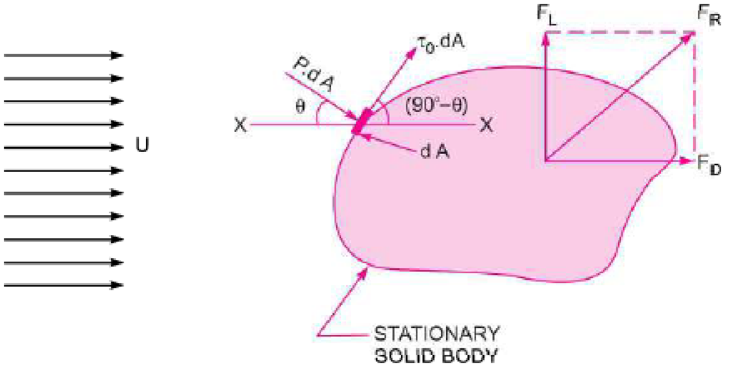
- **Flow past immersed bodies : Introduction**
- **Expression for DRAG and LIFT:**
- (b) Lift Force (F_L):
- The lift force on elemental area = Force due to pressure in the direction perpendicular to the direction of motion + Force due to shear stress in the direction perpendicular to the direction of motion.

$$= -pdA\sin\theta + \tau_0 dA\sin(90^\circ - \theta) = -pdA\sin\theta + \tau_0 dA\cos\theta$$

Negative sign is taken with pressure force as it is acting in the downward direction while shear force is acting vertically up.

Total lift,

$$F_L = \int \tau_0\, dA\cos\theta - \int pdA\sin\theta$$
$$= \int \tau_0 \cos\theta dA - \int p\sin\theta dA$$



- **Flow past immersed bodies : Introduction**
- **<u>Expression for DRAG and LIFT:</u>**
- The drag and lift for a body moving in a fluid of density ρ, at a uniform velocity U are calculated mathematically, as

$$F_D = C_D A \frac{\rho U^2}{2}$$

$$F_L = C_L A \frac{\rho U^2}{2}$$

C_D = Co-efficient of drag,
C_L = Co-efficient of lift,
A = Area of the body which is the projected area of the body perpendicular to the direction of flow

Then resultant force on the body, $F_R = \sqrt{F_D^2 + F_L^2}$

- The above equations which gives mathematical expression for drag and lift are derived by the method of dimensional analysis.

- **Flow past immersed bodies : Introduction**
- **<u>Dimensional Analysis of DRAG and LIFT:</u>**
- The drag force, F, on a smooth sphere depends on the relative velocity, V, the sphere diameter, D, the fluid density, ρ, and the fluid viscosity, μ.
- Obtain a set of dimensionless groups that can be used to correlate experimental data.

$$F = f(\rho, V, D, \mu) \text{ for a smooth sphere.}$$

① $F \quad V \quad D \quad \rho \quad \mu$ $\qquad n = 5$ dimensional parameters

② Select primary dimensions M, L, and t.

③

F	V	D	ρ	μ
$\frac{ML}{t^2}$	$\frac{L}{t}$	L	$\frac{M}{L^3}$	$\frac{M}{Lt}$

$r = 3$ primary dimensions

- **Flow past immersed bodies : Introduction**
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$$F = f(\rho, V, D, \mu) \text{ for a smooth sphere.}$$

④ Select repeating parameters ρ, V, D. $m = r = 3$ repeating parameters

⑤ Then $n - m = 2$ dimensionless groups will result. Setting up dimensional equations, we obtain

$$\Pi_1 = \rho^a V^b D^c F \quad \text{and} \quad \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b (L)^c \left(\frac{ML}{t^2}\right) = M^0 L^0 t^0$$

Equating the exponents of M, L, and t results in

$$\begin{aligned} M&: \quad a + 1 = 0 & a &= -1 \\ L&: \quad -3a + b + c + 1 = 0 & c &= -2 \\ t&: \quad -b - 2 = 0 & b &= -2 \end{aligned} \qquad \text{Therefore, } \Pi_1 = \frac{F}{\rho V^2 D^2}$$

- **Flow past immersed bodies : Introduction**
- **<u>Dimensional Analysis of DRAG and LIFT:</u>**
- The drag force, F, on a smooth sphere depends on the relative velocity, V, the sphere diameter, D, the fluid density, ρ, and the fluid viscosity, μ.
- Obtain a set of dimensionless groups that can be used to correlate experimental data.

$$F = f(\rho, V, D, \mu) \text{ for a smooth sphere.}$$

Similarly,

$$\Pi_2 = \rho^d V^e D^f \mu \quad \text{and} \quad \left(\frac{M}{L^3}\right)^d \left(\frac{L}{t}\right)^e (L)^f \left(\frac{M}{Lt}\right) = M^0 L^0 t^0$$

$$\begin{array}{lll} M: & d + 1 = 0 & d = -1 \\ L: & -3d + e + f - 1 = 0 & f = -1 \\ t: & -e - 1 = 0 & e = -1 \end{array} \quad \text{Therefore, } \Pi_2 = \frac{\mu}{\rho V D}$$

- **Flow past immersed bodies : Introduction**
- <u>**Dimensional Analysis of DRAG and LIFT:**</u>
- The drag force, F, on a smooth sphere depends on the relative velocity, V, the sphere diameter, D, the fluid density, ρ, and the fluid viscosity, μ.
- Obtain a set of dimensionless groups that can be used to correlate experimental data.

$$F = f(\rho, V, D, \mu) \text{ for a smooth sphere.}$$

The functional relationship is $\Pi_1 = f(\Pi_2)$, or

$$\frac{F}{\rho V^2 D^2} = f\left(\frac{\mu}{\rho V D}\right)$$

- **Flow past immersed bodies : Introduction**
- **<u>Dimensional Analysis of DRAG and LIFT:</u>**
- The drag force, F, on a smooth sphere depends on the relative velocity, V, the sphere diameter, D, the fluid density, ρ, and the fluid viscosity, μ.

Application of the Buckingham Pi theorem resulted in two dimensionless Π parameters that were written in functional form as

$$\frac{F_D}{\rho V^2 d^2} = f_2\left(\frac{\rho V d}{\mu}\right)$$

Note that d^2 is proportional to the cross-sectional area ($A = \pi d^2/4$) and therefore we could write

$$\frac{F_D}{\rho V^2 A} = f_3\left(\frac{\rho V d}{\mu}\right) = f_3(Re)$$

- Although we obtained equation for a sphere, the form of the equation is valid for incompressible flow over any body; the characteristic length used in the Reynolds number depends on the body shape.

- **Flow past immersed bodies : Introduction**
- **<u>Variation of Drag coefficient with Reynolds number:</u>**
- The drag force, F, on a smooth sphere depends on the relative velocity, V, the sphere diameter, D, the fluid density, ρ, and the fluid viscosity, μ.

The *drag coefficient*, C_D, is defined as

$$C_D \equiv \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

- The number 1/2 has been inserted (as was done in the defining equation for the friction factor) to form the familiar dynamic pressure.
- Then the equation can be written as:

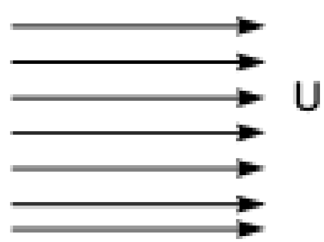
$$C_D = f(Re)$$

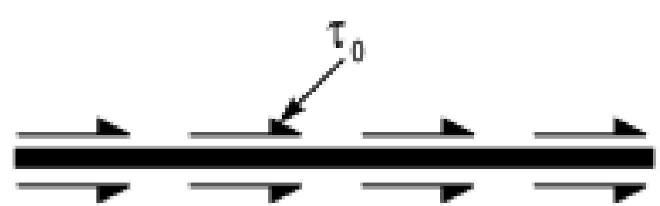
- **Flow past immersed bodies : Introduction**
- **<u>Flow over a Flat Plate Parallel to the Flow: Friction Drag:</u>**
- The total drag on a body is given by the equation.

$$F_D = \int p \cos \theta dA + \int \tau_0 \sin \theta dA$$

where $\int p \cos \theta dA$ = Pressure drag or form drag, and
$\int \tau_0 \sin \theta dA$ = Friction drag or skin drag or shear drag

- Consider the flow of fluid over a flat plate when the plate is placed parallel to the direction of the flow.
- In this cos θ, which is the angle made by pressure with the direction of motion will be 90°.
- Thus the term int. p cos θ dA will be zero and hence total drag will be equal to friction drag.





- **Flow past immersed bodies : Introduction**
- **Flow over a Flat Plate Parallel to the Flow: Friction Drag:**
- Thus the total drag equation is.

$$F_D = \int_{\text{plate surface}} \tau_w \, dA$$

- Substitute the total drag equation in the drag coefficient equation

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} = \frac{\int_{PS} \tau_w \, dA}{\frac{1}{2}\rho V^2 A}$$

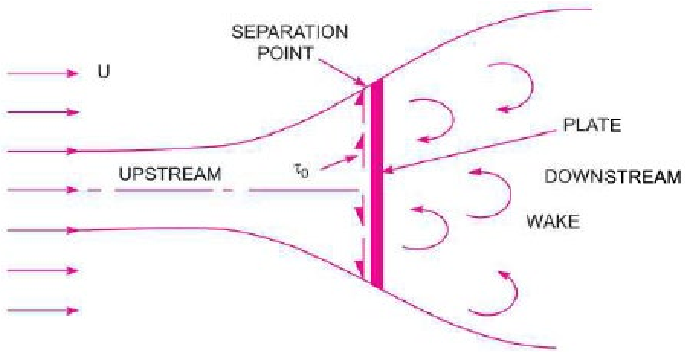
- The drag coefficient for a flat plate parallel to the flow depends on the **shear stress distribution** along the plate.

- **Flow past immersed bodies : Introduction**
- **<u>Flow over a Flat Plate Parallel to the Flow: Pressure Drag:</u>**
- If the plate is placed perpendicular to the flow, the angle θ made by the pressure with the direction of motion will be zero.
- Hence the term int. τ sin θ dA will become to zero and hence the total drag will be due to the pressure difference between the upstream and downstream side of the plate.

$$F_D = \int_{\text{surface}} p\,dA$$

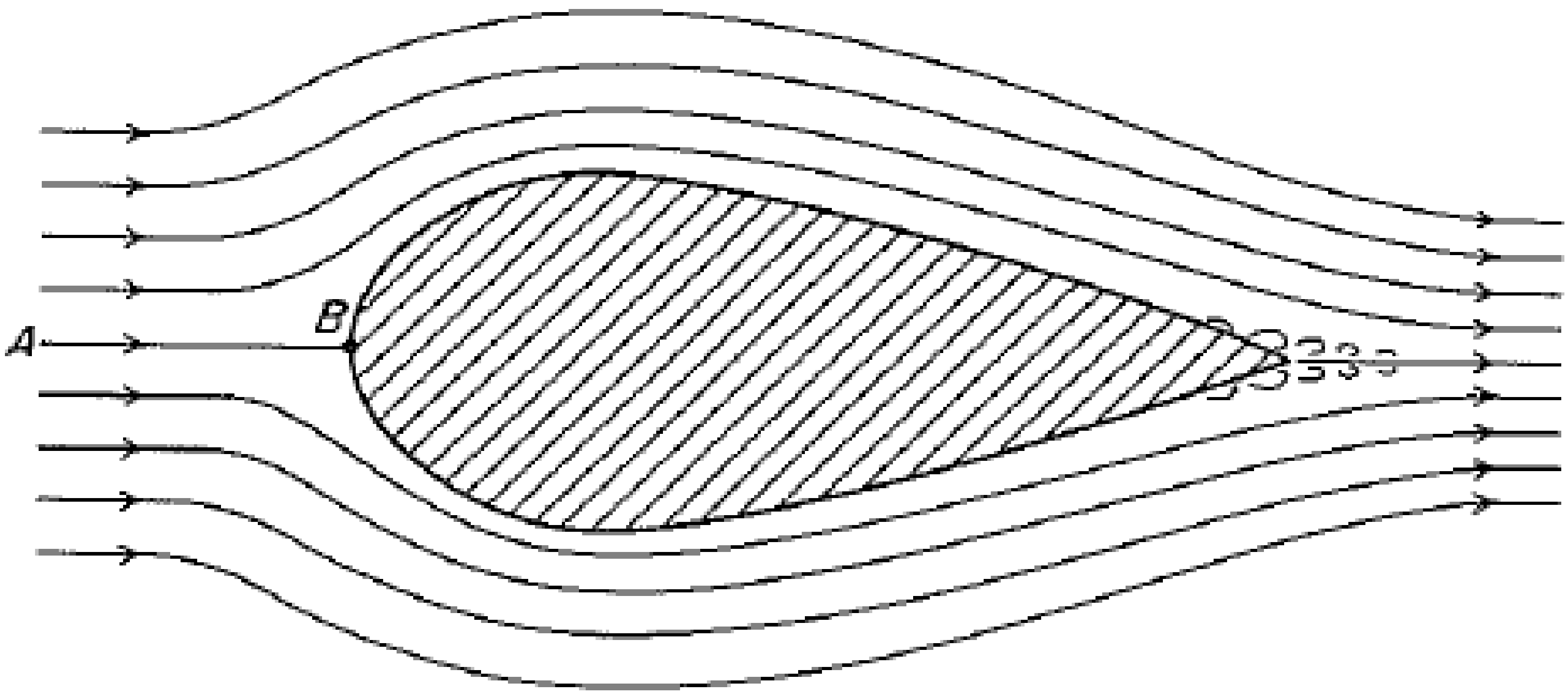
- If the plate is held at an angle with the direction of flow, both the terms will exist and total drag will be equal to the sum of pressure drag and friction drag.

$$F_D = \int p \cos\theta dA + \int \tau_0 \sin\theta dA$$



- **Flow past immersed bodies : Introduction**
- **<u>Streamlining:</u>**
- A stream-lined body is defined as that body whose surface coincides with the stream-lines, when the body is placed in a flow.
- In that case the separation of flow will take place only at the trailing edge (or rearmost part of the body).
- Though the boundary layer will start at the leading edge, will become turbulent from laminar, yet it does not separate upto the rearmost part of the body in the case of stream-lined body.
- Thus behind a stream-lined body, wake formation zone will be very small and consequently the pressure drag will be very small.
- Then the total drag on the stream-lined body will be due to friction only.
- A body may be stream-lined:
- 1. at low velocities but may not be so at higher velocities
- 2. when placed in a particular position in the flow but may not be so when placed in another position.

- **Flow past immersed bodies : Introduction**
- **<u>Streamlining:</u>**
- Pressure drag can be minimized by forcing separation towards the rear of the body.
- This is accomplished by streamlining.
- A perfectly streamlined body would have no wake and little or no pressure drag.



- **Flow past immersed bodies : Introduction**

Problem 14.1 *A flat plate 1.5 m × 1.5 m moves at 50 km/hour in stationary air of density 1.15 kg/m³. If the co-efficients of drag and lift are 0.15 and 0.75 respectively, determine :*

(i) The lift force, *(ii) The drag force,*
(iii) The resultant force, and
(iv) The power required to keep the plate in motion.

Solution. Given :

Area of the plate, $A = 1.5 \times 1.5 = 2.25 \text{ m}^2$

Velocity of the plate, $U = 50 \text{ km/hr} = \frac{50 \times 1000}{60 \times 60} \text{ m/s} = 13.89 \text{ m/s}$

Density of air $\rho = 1.15 \text{ kg/m}^3$

Co-efficient of drag, $C_D = 0.15$

Co-efficient of lift, $C_L = 0.75$

(*i*) **Lift Force (F_L).** Using equation (14.4),

$$F_L = C_L A \times \frac{\rho U^2}{2} = 0.75 \times 2.25 \times \frac{1.15 \times 13.89^2}{2} \text{ N} = \mathbf{187.20 \text{ N. Ans.}}$$

(*ii*) **Drag Force (F_D).** Using equation (14.3),

- **Flow past immersed bodies : Introduction**

Problem 14.1 *A flat plate 1.5 m × 1.5 m moves at 50 km/hour in stationary air of density 1.15 kg/m³. If the co-efficients of drag and lift are 0.15 and 0.75 respectively, determine :*

(i) The lift force, *(ii) The drag force,*

(iii) The resultant force, and

(iv) The power required to keep the plate in motion.

$$F_D = C_D \times A \times \frac{\rho U^2}{2} = 0.15 \times 2.25 \times \frac{1.15 \times 13.89^2}{2} \text{ N} = \mathbf{37.44.\ N.\ Ans.}$$

(*iii*) **Resultant Force (F_R).** Using equation (14.5),

$$F_R = \sqrt{F_D^2 + F_L^2} = \sqrt{37.44^2 + 187.20^2} \text{ N}$$

$$= \sqrt{1400 + 35025} = \mathbf{190.85\ N.\ Ans.}$$

(*iv*) **Power Required to keep the Plate in Motion**

$$P = \frac{\text{Force in the direction of motion} \times \text{Velocity}}{1000} \text{ kW}$$

$$= \frac{F_D \times U}{1000} = \frac{37.425 \times 13.89}{1000} \text{ kW} = \mathbf{0.519\ kW.\ Ans.}$$

- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**

- **<u>Reference:</u>**
- Fluid Mechanics by Fox
- Fluid Mechanics by Bansal
- Fluid Mechanics by Young
- NPTEL

CL203 – FLUID MECHANICS

III Semester BTech (Chemical Engineering)

CL203 FLUID MECHANICS

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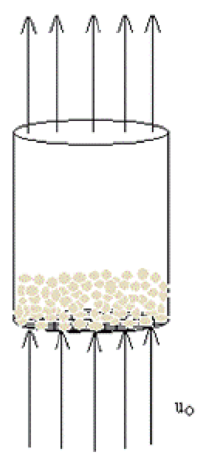
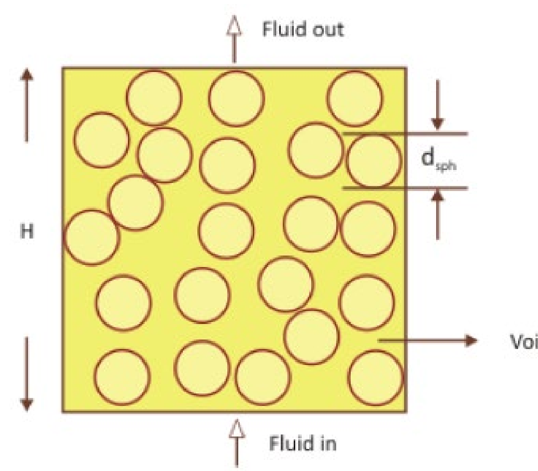
Lecture VIII

Turbine Design/scale up, Flow number, Power Requirement.

- **Packed Bed:**
- Chemical engineering operations commonly involve the use of packed fixed and fluidized beds.
- These are equipment in which a large surface area for contact between a liquid and a gas (absorption, distillation) or a solid and a gas or liquid (adsorption, catalysis) is obtained for achieving rapid mass and heat transfer.
- Packed beds of solid particles are used to in absorption, adsorption and distillation columns to increase interfacial area of contact between gas and liquid.
- A typical packed bed is a cylindrical column that is filled with a suitable material.
- The liquid is distributed as uniformly as possible at the top of the column and flows downward, wetting the packing material.
- A gas is admitted at the bottom, and flows upward, contacting the liquid in a countercurrent fashion.





- **Packed Bed:**
- Equivalent diameter and Sphericity:
- The Equivalent spherical diameter of an irregularly shaped object is the diameter of a sphere of equivalent volume.
- The total surface area is the surface area per particle times the number of particles, but it is more convenient to base the calculation on the volume fraction particles in the bed and the surface-volume ratio for the particles.
- This ratio is $6/D_p$ for a sphere, since $S_p = \pi D_p 2$ and $v_p = (1/6)\pi D_p 3$.
- For other shapes or irregular particles, the equation for surface-volume ratio includes a sphericity, defined as the surface-volume ratio for a sphere of Diameter D_p divided by the surface-volume ratio for the particle whose nominal size is D_p.

$$\Phi_s = \text{sphericity} = \left(\frac{\text{surface} - \text{area of a sphere}}{\text{surface area of particle having the same volume as that of the sphere}} \right)$$

- **Packed Bed:**
- Equivalent diameter and Sphericity:

a' = Specific surface area of a particle

$$= \left(\frac{\text{surface area of a particle}}{\text{volume of a particle}}\right)$$

$$= \frac{\left(\pi\, d_{sPh}^2\right)/\phi_s}{\left(\pi\, d_{sPh}^3\right)/6} = \frac{6}{\phi_s\, d_{sph}}$$

where the subscript 'sph' refers to the sphere having the same volume as that of the particle

$$\Phi_s = (6/D_p)/(s_p/v_p)$$

$$\frac{s_p}{v_p} = \frac{6}{\Phi_s D_p}$$

- **Packed Bed:**
- Equivalent diameter and Sphericity: Void fraction:
- If the particles are porous, the pores are generally too small to permit any significant flow through them, so Є is taken to be the external void fraction of the bed and not the total porosity. Here Є is the porosity or void fraction.

Volume of reactor = V_R

Number of particles = N_p

- Volume of one particle = V_p
- Volume of all the particles = V_p * N_p = $V_{ALL\text{-}PARTICLES}$

$$Void\ fraction = \varepsilon = \frac{V_{VOIDS}}{V_R}$$

$$\varepsilon = \frac{V_R - V_{ALL-PARTICLES}}{V_R}$$

$$\varepsilon = \frac{V_R - V_P N_P}{V_R}$$

$$N_P = \frac{V_R\left(1-\varepsilon\right)}{V_P}$$

- **Packed Bed:**
- Flow through a packed bed can be regarded as fluid flow past some number of submerged objects.
- In this case, the objects are uniform spherical particles of diameter dp.
- When there is no flow through the packed bed, the net gravitational force (including bouyancy) acts downward.
- When flow begins upward, friction forces act upward and counterbalance the net gravitational force.
- For a high enough fluid velocity, the friction force is large enough to lift the particles. T
- his represents the onset of fluidization.
- The frictional force can be expressed in terms of a friction factor.
- This leads to equations describing the flow of a fluid past a collection of particles.
- From the fluid mechanics perspective, the most important issue is that of the pressure drop required for the liquid or the gas to flow through the column at a specific flow rate.
- The Ergun equation is one such equation.

$$\Delta P = \frac{150 \mu L u_\infty (1-\varepsilon)^2}{D_p^2 \, \varepsilon^3} + \frac{1.75 \rho_f L u_\infty^2 (1-\varepsilon)}{D_p \, \varepsilon^3}$$

- **Packed Bed:**
- To calculate this quantity, we proceed as follows:
- We start, as always, with the generalized energy equation derived previously,

$$\frac{\Delta P}{\rho} + \Delta\left(\frac{u^2}{2}\right) + g\Delta z + F + w = 0$$

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

$\Delta\left(\frac{u^2}{2}\right) = 0$; Entering fluid leaves the column at the same speed (ie., continuity principle).

$w = 0$; No work done by or on the fluid

$g\Delta z = g\ L$; change in elevation ($z_{outlet} - z_{inlet}$) equals the total length of the column.

Substituting yields:

$$\frac{\Delta P}{\rho} + gL = -F$$

- **Packed Bed:**
- To calculate this quantity, we proceed as follows:
- We start, as always, with the generalized energy equation derived previously,

Also, previously we determined for flow in a pipe that:

$$F = 2 f_F {u_m}^2 \frac{L}{D}$$

Ff = ΔP.D/(2.L.ρ.V2)

Now, let's further simplify the system by making these assumptions:

1. For a horizontal bed or small L, gravity effect can be neglected.
2. Particles pack uniformly (ie., in stacks) resulting with continuous flow channels.
3. Bed can be modeled as a bundle of small pipes.
4. Flow is laminar (ie., $f_F = \frac{16}{Re}$)

- **Packed Bed:**

With these assumptions, the above two equations can be combined to give:

$$-\frac{\Delta P}{\rho} = 2\left(\frac{16}{Re}\right)\left(\frac{L}{D}\right){u_m}^2 = 2\left(\frac{16\mu}{\rho_f u_m D}\right)\left(\frac{L}{D}\right){u_m}^2 = \frac{32\mu L u_m}{\rho_f D^2}$$

- Now, the question is what velocity and diameter do we use?
- Should we use the 'superficial velocity' (the velocity that we cannot really see but can calculate it using the continuity equation as the overall volumetric flow rate divided by the overall column diameter), or should we use the actual velocity of the fluid travelling in the empty spaces between the packing particles?
- The packed bed is not a straight empty pipe, but rather a 'bundle of pipes' of irregular shapes. Therefore, we have the overall diameter pipe and the actual 'hydraulic diameter' (diameter in actual contact with the fluid), which one should we use?
- The answers to the above hypothetical questions are obviously: actual velocity and actual diameter; and they are determined as follows:

- **Packed Bed:**

Velocity

$$u_m = \frac{Q}{A_\varepsilon} = \frac{volumetric\ flow\ rate\ through\ the\ bed}{cross-sectional\ area\ of\ empty\ space} = \frac{Q}{\varepsilon A}$$

Where ε is the void fraction defined as:

$$\varepsilon = \frac{empty\ volume}{total\ volume\ (empty\ volume + solids\ volume)} = \frac{V_\varepsilon}{V_T} = \frac{V_\varepsilon}{V_\varepsilon + V_s}$$

And,

$$u_\infty = \frac{Q}{A} = superficial\ velocity$$

Substituting yields:

$$u_m = \frac{u_\infty}{\varepsilon}$$

- **Packed Bed:**

And the hydraulic radius is defined as:

$$D_h = \frac{4 * cross - sectional\ area\, for\ flow}{wetted\ perimeter}$$

Multiplying by L/L, gives:

$$D_h = \frac{4 * fluid\ volume\ (V_e)}{wetted\ surface\ area}$$

πd = Perimeter = P

wetted area = $\pi d \times L$

$$V_e = \varepsilon V_T = \varepsilon(V_e + V_s)$$

Solving for V_e, yields:

$$V_e = \frac{\varepsilon}{1-\varepsilon} V_s$$

Where $V_s = \#\ of\ particles * V_p = N_p * \frac{\pi}{6} D_p^3$

- **Packed Bed:**

Wetted surface area = total solids surface area = $N_p * \pi D_p^2$

Substituting above for D_h, gives:

$$D_h = \frac{4 * \frac{\varepsilon}{1-\varepsilon} N_p * \frac{\pi}{6} D_p^3}{N_p * \pi D_p^2}$$

- **Rearranging**

$$D_h = \frac{4\varepsilon D_p}{6\,(1-\varepsilon)}$$

Finally, substituting above for the pressure drop, yields:

$$\Delta P = \frac{72\mu L u_\infty (1-\varepsilon)^2}{D_p^2\, \varepsilon^3}$$

- **Packed Bed:**

$$\Delta P = \frac{72\mu L u_{\infty}(1-\varepsilon)^2}{D_p^2\ \varepsilon^3}$$

- The real situation is that the flow flows in a tortuous path (not straight as was initially assumed), and bed length can not really be ignored, therefore, experimental results has consistently resulted with data suggesting replacing the constant 72 by 150:
- Experimental data shows, we need to multiply the pressure requirement by ~ 2 (exactly 100/48)

$$\boxed{\Delta P = \frac{150\mu L u_{\infty}(1-\varepsilon)^2}{D_p^2\ \varepsilon^3}}$$

This is known as the Blake-Kozeny equation. (best use for $\varepsilon < 0.5$ and $Re_p = \frac{\rho_f u_{\infty} D_p}{(1-\varepsilon)\mu} < 10$).

- **Packed Bed:**
- For Turbulent Flow:

  - Pressure drop and shear stress equations

$$Force = \Delta P \frac{\pi D^2}{4} \varepsilon \qquad Force = \tau \text{ Contact area}$$

  - For high turbulence (high Re),

$$f = \frac{\tau}{\frac{1}{2}\rho V_{avg}^2} = \text{constant} \qquad \therefore \tau = \text{constant } \tfrac{1}{2}\rho V_{avg}^2$$

- Can we relate average velocity and superficial velocity?

  - However $V_{avg} = \frac{V_0}{\varepsilon}$

$$\tau = K\rho \frac{V_0^2}{\varepsilon^2}$$

- **Packed Bed:**
- For Turbulent Flow:

- We have already developed an expression for contact area

$$\text{Wetted Area} = N_p A_p = \frac{V_R(1-\varepsilon)}{V_p} A_p = V_R(1-\varepsilon)\frac{A_p}{V_p}$$

- Hence, force balance

$$Force = \Delta P \frac{\pi D^2}{4}\varepsilon = \tau \text{ Contact area}$$

$$= \left(K\rho\frac{V_0^2}{\varepsilon^2}\right)\left(V_R(1-\varepsilon)\frac{A_p}{V_p}\right)$$

- Volume of reactor (say, height of bed = L)

$$V_R = \frac{\pi D^2}{4} L$$

$$\Delta P = \left(K\rho\frac{V_0^2}{\varepsilon^3}\right)\left(L(1-\varepsilon)\frac{A_p}{V_p}\right)$$

- **Packed Bed:**
- For Turbulent Flow:

$$\Delta P = \left(K \rho \frac{V_0^2}{\varepsilon^3} \right) \left(L (1 - \varepsilon) \frac{6}{D_p} \right)$$

- Value of K based on experiments ~ 7/24

$$\Delta P = \frac{1.75 \rho_f L u_\infty^2 (1 - \varepsilon)}{D_p \, \varepsilon^3}$$

This equation is known as the Burke-Plummer Equation.

- **Packed Bed:**

For the Transition region, the Ergun equation is to be used:

$$\Delta P = \frac{150 \mu L u_\infty (1-\varepsilon)^2}{D_p^2\, \varepsilon^3} + \frac{1.75 \rho_f L u_\infty^2 (1-\varepsilon)}{D_p\, \varepsilon^3}$$

The Ergun equation is commonly expressed as follows in different references:

$$\frac{\Delta P}{\rho_f u_\infty^2} \frac{D_p}{L} \frac{\varepsilon^3}{(1-\varepsilon)} = \frac{150}{Re_p} + 1.75$$

Note:

- The above equation can be used with gases using average gas density between inlet and outlet
- Flow turbulent flow, the 1st term on the RHS vanishes
- For laminar flow, the 2nd term on the RHS can be ignored

Therefore, to solve problems, one could just start with the Ergun equation, and can either use one of the terms on the RHS depending on the flow regime (laminar or turbulent) or even use the equation as is with minimal change!

- **Packed Bed:**

Air (ρ = 1.22 Kg/m^3, μ = 1.9 X 10^{-5} pa.s) is flowing in a fixed bed of a diameter 0.5 m and height 2.5 m. The bed is packed with spherical particles of diameter 10 mm. The void fraction is 0.38. The air mass flow rate is 0.5 kg/s. Calculate the pressure drop across the bed of particles.

Solution

$$Q = \text{volumetric flow rate} = \frac{0.5}{1.22} = 0.41 \ \text{m}^3/\text{s}$$

$$A = \frac{\pi}{4}D^2 = \left(\frac{\pi}{4}\right)(0.5)^2 = 0.1963 \ \text{m}^2$$

$$u_\infty = \frac{Q}{A} = \frac{0.41}{0.1963} = 2.1 \ \text{m/s}$$

$$Re_p = \frac{\rho u_\infty D_p}{(1-\varepsilon)\mu} = \frac{(1.22)(2.1)(10 X 10^{-3})}{(1-0.38)\ (1.9 X 10^{-5})}$$

$$Re_p = 2174$$

$$f_p = \frac{150}{2174} + 1.75 = 1.819$$

- **Packed Bed:**

Air (ρ = 1.22 Kg/m^3, μ = 1.9 X 10^{-5} pa.s) is flowing in a fixed bed of a diameter 0.5 m and height 2.5 m. The bed is packed with spherical particles of diameter 10 mm. The void fraction is 0.38. The air mass flow rate is 0.5 kg/s. Calculate the pressure drop across the bed of particles.

$$\frac{D_p \varepsilon^3}{\rho u_\infty^2 (1-\varepsilon)} \frac{|\Delta P|}{L} = 1.819$$

$$\Delta P = \frac{(1.819)(1.22)\,(2.1)^2\,(1-0.38)(2.5)}{(10\,X 10^{-3})\,(0.38)^3}$$

$$\Delta P = 0.276 \text{ X } 10^5 \text{ pa.}$$

- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**

- **<u>Reference:</u>**
- Unit operations of Chemical Engineering By McCabe
- NPTEL