CL24203 – FLUID MECHANICS

III Semester BTech (Chemical Engineering)

CL203 FLUID MECHANICS

- **Module 3:**
- **Internal incompressible viscous flow:**
- **Introduction;**
- **flow of incompressible fluid in circular pipe;**
- **laminar flow for Newtonian fluid;**
- **Hagen-Poiseuille equation;**
- flow of Non-Newtonian fluid,
- introduction to turbulent flow in a pipe;
- energy consideration in pipe flow,
- **relation between average and maximum velocity,**
- Bernoulli’s equation–kinetic energy correction factor; head loss; friction factor; major and minor losses,
- Pipe fittings and valves. [8]

LECTURE PLAN AND LEARNING OBJECTIVES FOR 40 [ONE HOUR] LECTURES

Module 3: Internal incompressible viscous flow: Introduction; flow of incompressible fluid in circular pipe; laminar flow for Newtonian fluid; Hagen-Poiseuille equation; flow of Non-Newtonian fluid, introduction to turbulent flow in a pipe; energy consideration in pipe flow, relation between average and maximum velocity, Bernoulli's equation–kinetic energy correction factor; head loss; friction factor; major and minor losses, Pipe fittings and valves. [8]

Lecture I

Internal incompressible viscous flow: Introduction; flow of incompressible fluid in circular pipe;

Lecture II

Laminar flow for Newtonian fluid; Hagen-Poiseuille equation;

Lecture III

Flow of Non-Newtonian fluid,

Lecture IV

Introduction to turbulent flow in a pipe;

Lecture V

Energy consideration in pipe flow,

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Module 3: Internal incompressible viscous flow: Introduction; flow of incompressible fluid in circular pipe; laminar flow for Newtonian fluid; Hagen-Poiseuille equation; flow of Non-Newtonian fluid, introduction to turbulent flow in a pipe; energy consideration in pipe flow, relation between average and maximum velocity, Bernoulli's equation–kinetic energy correction factor; head loss; friction factor; major and minor losses, Pipe fittings and valves. [8]

Lecture VI

Relation between average and maximum velocity,

Lecture VII

Bernoulli's equation–kinetic energy correction factor;

Lecture VIII

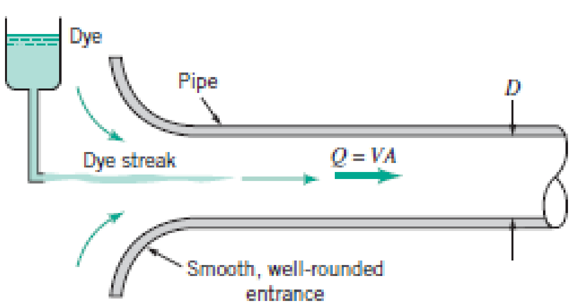
Head loss; friction factor; major and minor losses, Pipe fittings and valves.

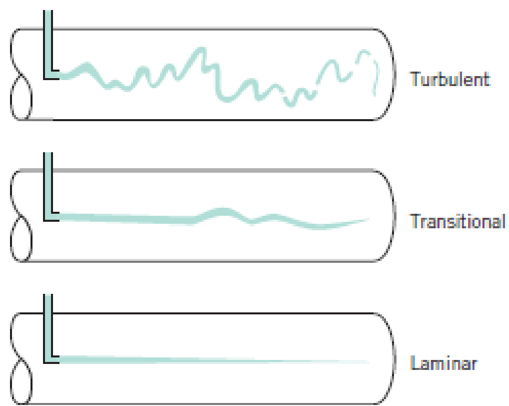
- **Internal incompressible viscous flow : Introduction**
- Flows completely bounded by solid surfaces are called internal flows.
- Thus internal flows include flows through pipes, ducts, nozzles, diffusers, sudden contractions and expansions, valves, and fittings.
- Internal flows may be laminar or turbulent.
- For internal flows, the flow regime (laminar or turbulent) is primarily a function of the Reynolds number.

$$R_e = \frac{\rho V D}{\mu}$$

where ρ = Density of fluid flowing through pipe
V = Average velocity of fluid
D = Diameter of pipe and
μ = Viscosity of fluid.

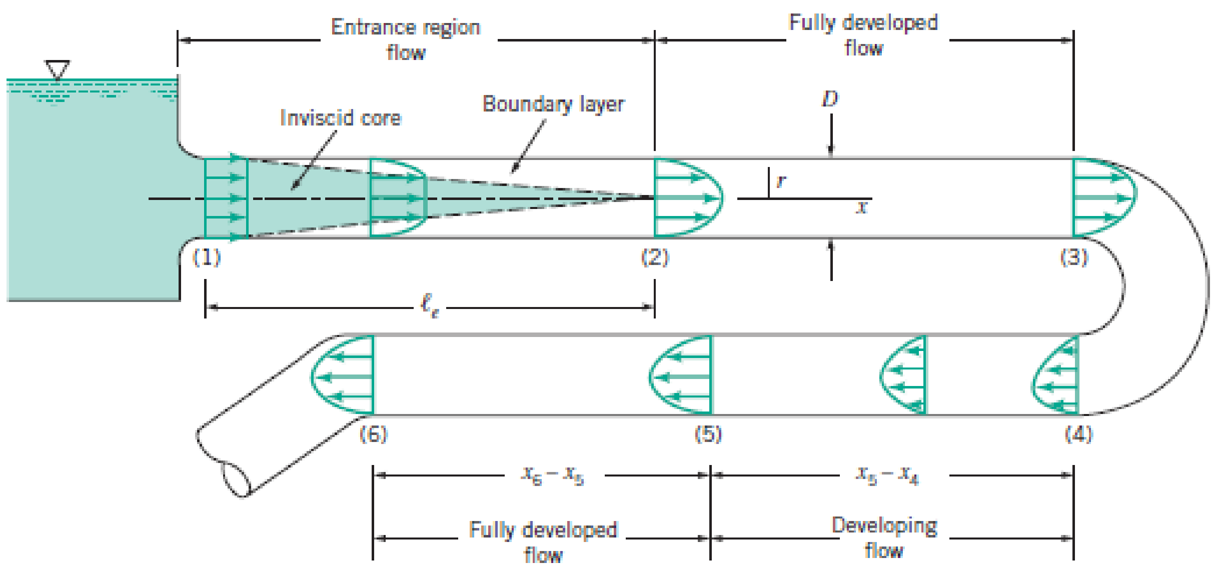
- **Internal incompressible viscous flow : Introduction**
- **Laminar or Turbulent Flow:**
- The flow of a fluid in a pipe may be laminar flow or it may be turbulent flow.
- Osborne Reynolds, a British scientist and mathematician, was the first to distinguish the difference between these two classifications of flow by using a simple apparatus.
- For "small enough flowrates" the dye streak (a streakline) will remain as a well defined line as it flows along, with only slight blurring due to molecular diffusion of the dye into the surrounding water.
- For a somewhat larger "intermediate flowrate" the dye streak fluctuates in time and space, and intermittent bursts of irregular behavior appear along the streak.
- However, for "large enough flowrates" the dye streak almost immediately becomes blurred and spreads across the entire pipe in a random fashion. These three characteristics, denoted as laminar, transitional, and turbulent flow, respectively





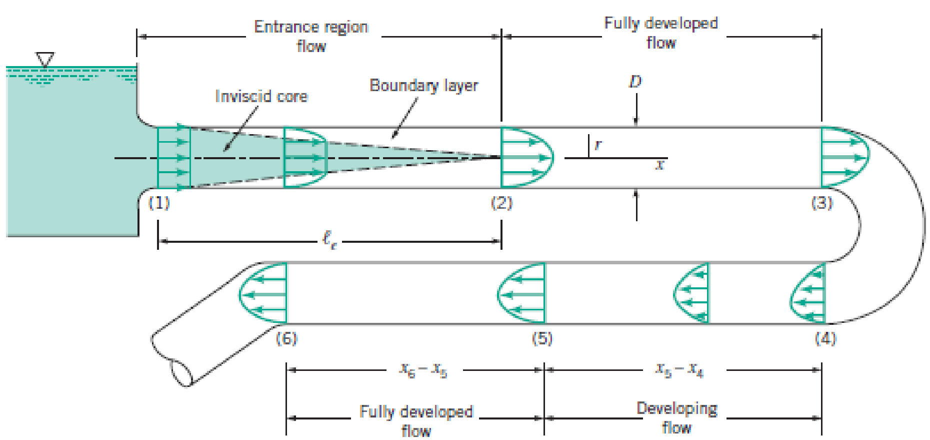
- **Internal incompressible viscous flow : Introduction**
- **Laminar or Turbulent Flow:**
- That is, the flow in a pipe is laminar, transitional, or turbulent provided the Reynolds number is “small enough,” “intermediate,” or “large enough.”
- It is not only the fluid velocity that determines the character of the flow—its density, viscosity, and the pipe size are of equal importance.
- These parameters combine to produce the Reynolds number.

- **Internal incompressible viscous flow : Introduction**
- **Entrance Region and Fully developed Flow:**
- Any fluid flowing in a pipe had to enter the pipe at some location.
- The region of flow near where the fluid enters the pipe is termed the entrance region.
- As shown, the fluid typically enters the pipe with a nearly uniform velocity profile at section (1).
- As the fluid moves through the pipe, viscous effects cause it to stick to the pipe wall (the no-slip boundary condition).
- This is true whether the fluid is relatively inviscid air or a very viscous oil.

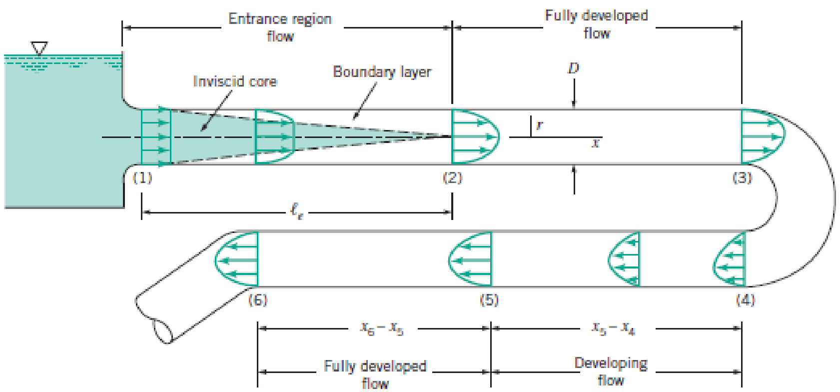


- **Internal incompressible viscous flow : Introduction**
- **Entrance Region and Fully developed Flow:**
- Thus, a boundary layer in which viscous effects are important is produced along the pipe wall such that the initial velocity profile changes with distance along the pipe, x, until the fluid reaches the end of the entrance length, section (2), beyond which the velocity profile does not vary with x.
- The boundary layer has grown in thickness to completely fill the pipe.
- The shape of the velocity profile in the pipe depends on whether the flow is laminar or turbulent, as does the entrance length, l_e. Typical entrance lengths are given by

$$\frac{\ell_e}{D} = 0.06\,\text{Re for laminar flow} \qquad \frac{\ell_e}{D} = 4.4\,(\text{Re})^{1/6}\text{ for turbulent flow}$$

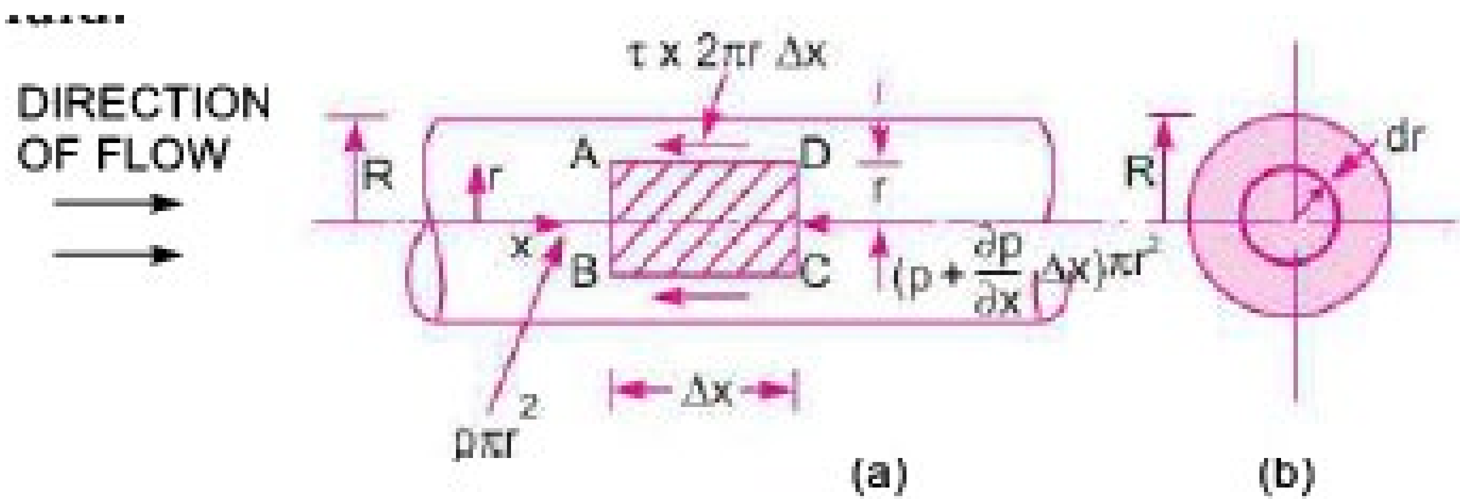


- **Internal incompressible viscous flow : Introduction**
- **Entrance Region and Fully developed Flow:**
- Once the fluid reaches the end of the entrance region, section (2) of Fig, the flow is simpler to describe because the velocity is a function of only the distance from the pipe centerline, r, and independent of x.
- This is true until the character of the pipe changes in some way, such as a change in diameter or the fluid flows through a bend, valve, or some other component at section (3).
- The flow between (2) and (3) is termed **fully developed flow**.
- Beyond the interruption of the fully developed flow [at section (4)], the flow gradually begins its return to its fully developed character [section (5)] and continues with this profile until the next pipe system component is reached [section (6)].



- **Flow of incompressible fluid in Circular Pipe:**
- Let us consider the fully developed laminar flow in a horizontal pipe of radius R.
- The viscous fluid is flowing from left to right in the pipe as shown in the Fig.
- Consider a fluid element of radius r, sliding in a cylindrical fluid element of radius (r + dr).
- Let the length of the fluid element be Δx.
- If "p" the intensity of pressure on the face AB.
- The intensity of pressure on face CD will be

$$\left(p + \frac{\partial p}{\partial x}\Delta x\right)$$



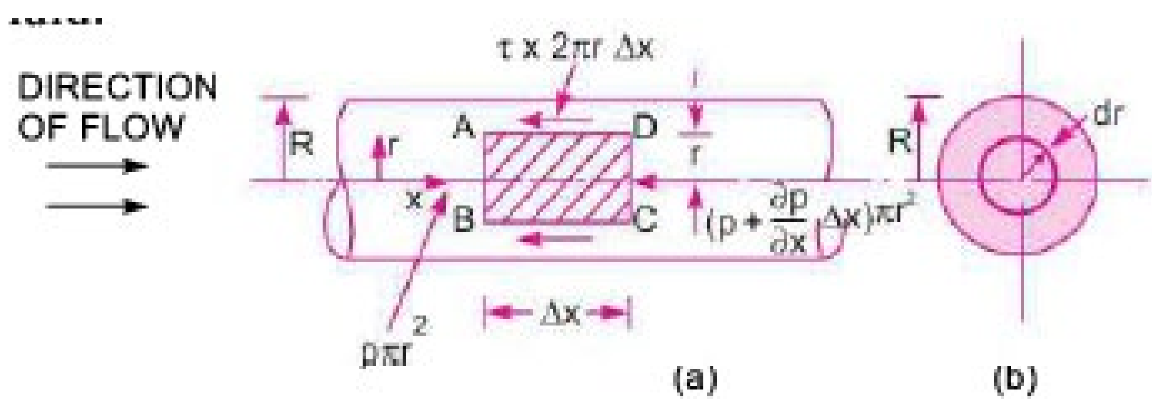
- **Flow of incompressible fluid in Circular Pipe:**
- Let us consider the fully developed laminar flow in a horizontal pipe of radius R.
- Then the forces act on the fluid element are:

The pressure force, $p \times \pi r^2$ on face AB.

The pressure force, $\left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2$ on face CD.

- As well as, the shear force on the surface of fluid element,

$$\tau \times 2\pi r \Delta x$$

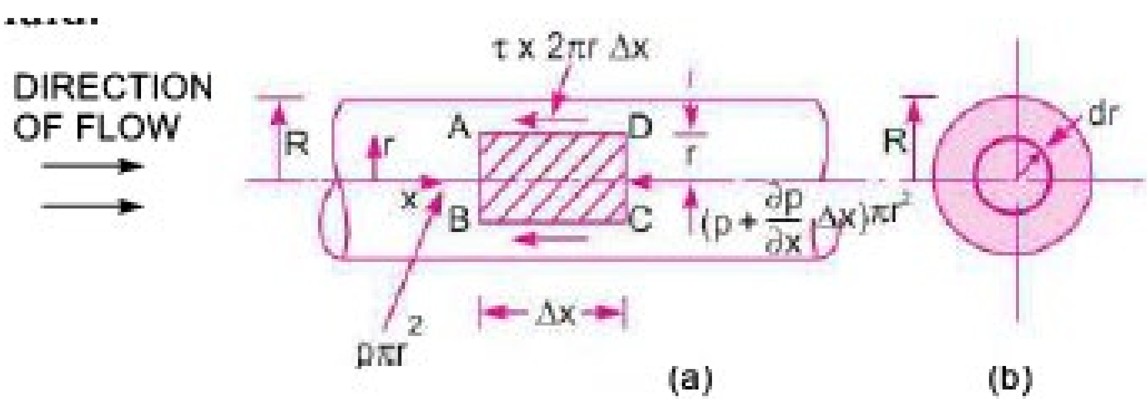


- **Flow of incompressible fluid in Circular Pipe:**
- Let us consider the fully developed laminar flow in a horizontal pipe of radius R.
- The summation of all forces in the direction of flow must be zero ie.,

$$p\pi r^2 - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

$$-\frac{\partial p}{\partial x}\Delta x \pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

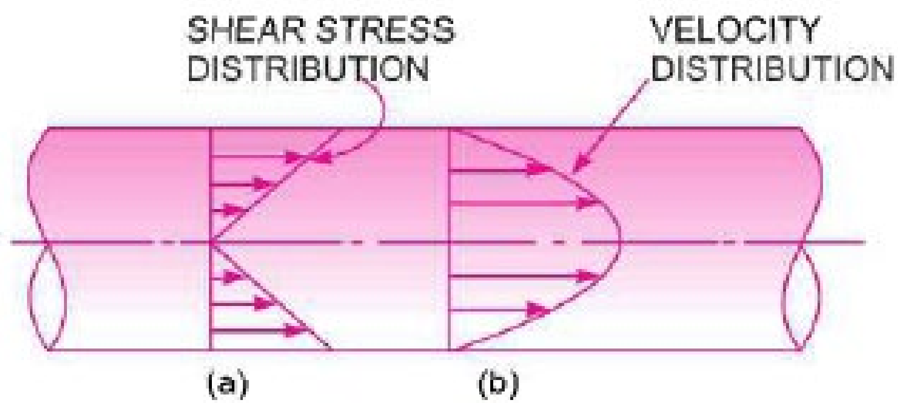
$$-\frac{\partial p}{\partial x}\,.\,r - 2\tau = 0 \qquad\qquad \tau = -\frac{\partial p}{\partial x}\frac{r}{2}$$

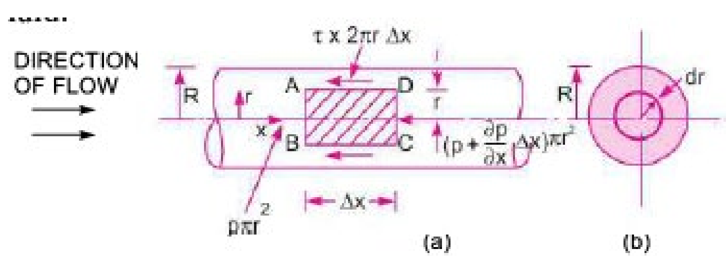


- **Flow of incompressible fluid in Circular Pipe:**
- Let us consider the fully developed laminar flow in a horizontal pipe of radius R.
- From the below equation, we will get that the shear stress ζ across the section varies with ‘r’ as δp/δx across a section is constant.

$$\tau = -\frac{\partial p}{\partial x}\frac{r}{2}$$

- Hence the shear stress distribution across a section is linear as shown in the Fig.





- **Flow of incompressible fluid in Circular Pipe:**
- (i) Velocity Distribution:
- To obtain the velocity distribution across a section, the value of shear stress is substituted in the equation

$$\tau = \mu \frac{du}{dy}$$

- But the ‘y’ is measured from the pipe wall.
- Hence

$$y = R - r \quad \text{and} \quad dy = -dr$$

$$\tau = \mu \frac{du}{-dr} = -\mu \frac{du}{dr}$$

- Substitute the above value in the equation.

$$-\mu \frac{du}{dr} = -\frac{\partial p}{\partial x}\frac{r}{2} \quad \text{or} \quad \frac{du}{dr} = \frac{1}{2\mu}\frac{\partial p}{\partial x}r$$

- **Flow of incompressible fluid in Circular Pipe:**
- (i) Velocity Distribution:
- Integrate the above equation w.r.t 'r', we have:

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 + C$$

- Where C is the constant of integration and its value is obtained from the boundary condition that at r = R and u = 0.

$$0 = \frac{1}{4\mu}\frac{\partial p}{\partial x} R^2 + C$$

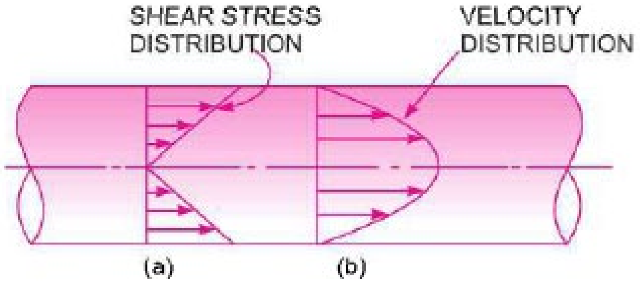
$$C = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

- **Flow of incompressible fluid in Circular Pipe:**
- (i) Velocity Distribution:
- Substitute the value of C in the equation, we get

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 - \frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2]$$

- In the above equation, value of μ, δp/δx and R are constant, which means the velocity 'u' varies with the square of 'r'.
- Thus the above equation is a equation of parabola.
- This shows that the velocity distribution across the section of a pipe is parabolic and the velocity distribution is shown in the Fig.



- **Flow of incompressible fluid in Circular Pipe:**
- (ii) Ratio of Maximum velocity to Average velocity:
- The velocity is maximum when r = 0 in the equation.

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x}\,r^2 - \frac{1}{4\mu}\frac{\partial p}{\partial x}\,R^2$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2]$$

- Thus the maximum velocity is obtained as:

$$U_{\max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x}\,R^2$$

- **Flow of incompressible fluid in Circular Pipe:**
- (ii) Ratio of Maximum velocity to Average velocity:
- The average velocity is obtained by dividing the discharge of the fluid across the section by the area of the pipe.
- The discharge 'Q' across the section is obtained by considering the flow through a circular ring element of radius r and thickness dr.
- The fluid flowing per second through the elementary ring

$$dQ = \text{velocity at a radius } r \times \text{area of ring element}$$

$$= u \times 2\pi r\, dr$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2] \times 2\pi r\, dr$$

$$Q = \int_0^R dQ = \int_0^R -\frac{1}{4\mu}\frac{\partial p}{\partial x}\,(R^2 - r^2) \times 2\pi r\, dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \int_0^R (R^2 - r^2)\, r dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \int_0^R (R^2 r - r^3)\, dr$$

- **Flow of incompressible fluid in Circular Pipe:**
- (ii) Ratio of Maximum velocity to Average velocity:
- The average velocity is obtained by dividing the discharge of the fluid across the section by the area of the pipe.

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R = \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \times \frac{R^4}{4} = \frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^4$$

$\therefore$ Average velocity,
$$\bar{u} = \frac{Q}{\text{Area}} = \frac{\frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^4}{\pi R^2}$$

$$\bar{u} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^2$$

- **Flow of incompressible fluid in Circular Pipe:**
- (ii) Ratio of Maximum velocity to Average velocity:
- Divide the equation of maximum velocity to average velocity,

$$\frac{U_{max}}{\bar{u}} = \frac{-\frac{1}{4\mu}\frac{\partial p}{\partial x}R^2}{\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2} = 2.0$$

- Therefore the ratio of maximum velocity to average velocity for a circular pipe is 2.

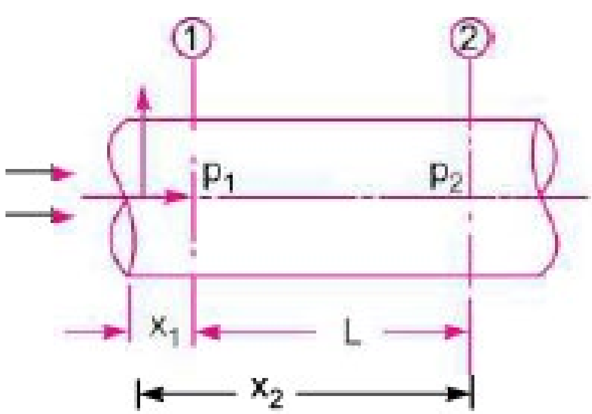
- $U_{max} = 2\ U_{avg}$

- **Flow of incompressible fluid in Circular Pipe:**
- (iii) Pressure drop for a given Length (L) of a pipe:
- From the average velocity equation, we have:

$$\bar{u} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^2 \quad \text{or} \quad \left(\frac{-\partial p}{\partial x}\right) = \frac{8\mu \bar{u}}{R^2}$$

- Integrating the above equation w.r.t 'x' we get:

$$-\int_2^1 dp = \int_2^1 \frac{8\mu \bar{u}}{R^2}\, dx$$



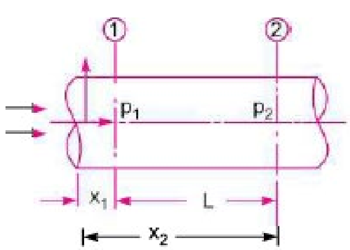
- **Flow of incompressible fluid in Circular Pipe:**
- (iii) Pressure drop for a given Length (L) of a pipe:

$$-[p_1 - p_2] = \frac{8\mu\bar{u}}{R^2}[x_1 - x_2] \text{ or } (p_1 - p_2) = \frac{8\mu\bar{u}}{R^2}[x_2 - x_1]$$

$$= \frac{8\mu\bar{u}}{R^2} L \qquad \{\because \quad x_2 - x_1 = L \text{ from Fig}$$

$$= \frac{8\mu\bar{u}L}{(D/2)^2} \qquad \left\{\because \quad R = \frac{D}{2}\right\}$$

$$(p_1 - p_2) = \frac{32\mu\bar{u}L}{D^2},$$ where $p_1 - p_2$ is the drop of pressure.



- **Flow of incompressible fluid in Circular Pipe:**
- (iii) Pressure drop for a given Length (L) of a pipe:

$$\therefore \quad \text{Loss of pressure head} = \frac{p_1 - p_2}{\rho g}$$

then pressure at A

$$= \rho \times g \times h \ \frac{\text{N}}{\text{m}^2}.$$

$$\therefore \quad \frac{p_1 - p_2}{\rho g} = h_f = \frac{32\mu \bar{u} L}{\rho g D^2}$$

$$(p_1 - p_2) = \frac{32\mu \bar{u} L}{D^2}$$

- Substitute D = 2R in the above equation, we get:
- (p1 – p2)/L = ((8µU)/R2) where p1 – p2 = P
- P/L = ((8µU)/R2) We know that Q = U A; U = Q/A and A = πR2
- P/L = ((8µQ)/ πR2 . R2)

$$Q = \frac{\pi R^4}{8\mu} \frac{p}{L}$$

- This equation is known as Hagen Poiseuille formula.

- **Flow of incompressible fluid in Circular Pipe:**
- (iii) Pressure drop for a given Length (L) of a pipe:

Problem 9.1 *A crude oil of viscosity 0.97 poise and relative density 0.9 is flowing through a horizontal circular pipe of diameter 100 mm and of length 10 m. Calculate the difference of pressure at the two ends of the pipe, if 100 kg of the oil is collected in a tank in 30 seconds.*

Solution. Given : $\mu = 0.97 \text{ poise} = \frac{0.97}{10} = 0.097 \text{ Ns/m}^2$

Relative density $= 0.9$

$\therefore$ ρ_0, or density, $= 0.9 \times 1000 = 900 \text{ kg/m}^3$

Dia. of pipe, $D = 100 \text{ mm} = 0.1 \text{ m}$

$L = 10 \text{ m}$

Mass of oil collected, $M = 100 \text{ kg}$

Time, $t = 30 \text{ seconds}$

Calculate difference of pressure or $(p_1 - p_2)$.

The difference of pressure $(p_1 - p_2)$ for viscous or laminar flow is given by

$$p_1 - p_2 = \frac{32\mu \bar{u} L}{D^2}, \text{ where } \bar{u} = \text{average velocity} = \frac{Q}{\text{Area}}$$

- **Flow of incompressible fluid in Circular Pipe:**
- (iii) Pressure drop for a given Length (L) of a pipe:

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Now, mass of oil/sec $= \frac{100}{30}$ kg/s

$= \rho_0 \times Q = 900 \times Q \qquad (\because \rho_0 = 900)$

$\therefore \qquad \frac{100}{30} = 900 \times Q$

$\therefore \qquad Q = \frac{100}{30} \times \frac{1}{900} = 0.0037 \text{ m}^3/\text{s}$

$$\bar{u} = \frac{Q}{\text{Area}} = \frac{.0037}{\frac{\pi}{4} D^2} = \frac{.0037}{\frac{\pi}{4}(.1)^2} = 0.471 \text{ m/s.}$$

- **Flow of incompressible fluid in Circular Pipe:**
- (iii) Pressure drop for a given Length (L) of a pipe:

Problem 9.1 *A crude oil of viscosity 0.97 poise and relative density 0.9 is flowing through a horizontal circular pipe of diameter 100 mm and of length 10 m. Calculate the difference of pressure at the two ends of the pipe, if 100 kg of the oil is collected in a tank in 30 seconds.*

For laminar or viscous flow, the Reynolds number (R_e) is less than 2000. Let us calculate the Reynolds number for this problem.

Reynolds number, $$R_e{}^* = \frac{\rho V D}{\mu}$$

where $\rho = \rho_0 = 900$, $V = \bar{u} = 0.471$, $D = 0.1$ m, $\mu = 0.097$

$$\therefore \qquad R_e = 900 \times \frac{.471 \times 0.1}{0.097} = 436.91$$

As Reynolds number is less than 2000, the flow is laminar.

$$\therefore \qquad p_1 - p_2 = \frac{32\mu\bar{u}L}{D^2} = \frac{32 \times 0.097 \times .471 \times 10}{(.1)^2} \text{ N/m}^2$$

$$= 1462.28 \text{ N/m}^2 = 1462.28 \times 10^{-4} \text{ N/cm}^2 = \mathbf{0.1462\ N/cm^2}. \textbf{ Ans.}$$

- **Flow of incompressible fluid in Circular Pipe:**
- (iii) Pressure drop for a given Length (L) of a pipe:

Problem 9.3 *A laminar flow is taking place in a pipe of diameter 200 mm. The maximum velocity is 1.5 m/s. Find the mean velocity and the radius at which this occurs. Also calculate the velocity at 4 cm from the wall of the pipe.*

Solution. Given : Dia. of pipe, $D = 200$ mm $= 0.20$ m

$$U_{max} = 1.5 \text{ m/s}$$

Find (*i*) Mean velocity, $\bar{u}$

(*ii*) Radius at which $\bar{u}$ occurs

(*iii*) Velocity at 4 cm from the wall.

(*i*) **Mean velocity, $\bar{u}$**

Ratio of $\dfrac{U_{max}}{\bar{u}} = 2.0$ or $\dfrac{1.5}{\bar{u}} = 2.0$ $\therefore$ $\bar{u} = \dfrac{1.5}{2.0} = $ **0.75 m/s. Ans.**

- **Flow of incompressible fluid in Circular Pipe:**
- (iii) Pressure drop for a given Length (L) of a pipe:

Problem 9.3 *A laminar flow is taking place in a pipe of diameter 200 mm. The maximum velocity is 1.5 m/s. Find the mean velocity and the radius at which this occurs. Also calculate the velocity at 4 cm from the wall of the pipe.*

(*ii*) **Radius at which** $\bar{u}$ occurs

The velocity, u, at any radius 'r' is given by (9.3)

$$u = -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2] = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2 \left[1 - \frac{r^2}{R^2}\right]$$

But from equation (9.4) U_{max} is given by

$$U_{max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2 \quad \therefore \quad u = U_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

- **Flow of incompressible fluid in Circular Pipe:**
- (iii) Pressure drop for a given Length (L) of a pipe:

Problem 9.3 *A laminar flow is taking place in a pipe of diameter 200 mm. The maximum velocity is 1.5 m/s. Find the mean velocity and the radius at which this occurs. Also calculate the velocity at 4 cm from the wall of the pipe.*

Now, the radius r at which $u = \bar{u} = 0.75$ m/s

$$\therefore \quad 0.75 = 1.5\left[1-\left(\frac{r}{D/2}\right)^2\right]$$

$$= 1.5\left[1-\left(\frac{r}{0.2/2}\right)^2\right] = 1.5\left[1-\left(\frac{r}{0.1}\right)^2\right]$$

$$\therefore \quad \frac{0.75}{0.50} = 1 - \left(\frac{r}{0.1}\right)^2$$

$$\therefore \quad \left(\frac{r}{0.1}\right)^2 = 1 - \frac{.75}{1.50} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\therefore \quad \frac{r}{0.1} = \sqrt{\frac{1}{2}} = \sqrt{0.5}$$

$$\therefore \quad r = 0.1 \times \sqrt{.5} = 0.1 \times .707 = .0707 \text{ m}$$

$$= \mathbf{70.7\ mm.\ Ans.}$$

- **Flow of incompressible fluid in Circular Pipe:**
- (iii) Pressure drop for a given Length (L) of a pipe:

Problem 9.3 *A laminar flow is taking place in a pipe of diameter 200 mm. The maximum velocity is 1.5 m/s. Find the mean velocity and the radius at which this occurs. Also calculate the velocity at 4 cm from the wall of the pipe.*

(*iii*) Velocity at 4 cm from the wall

$$r = R - 4.0 = 10 - 4.0 = 6.0 \text{ cm} = 0.06 \text{ m}$$

$\therefore$ The velocity at a radius = 0.06 m

or 4 cm from pipe wall is given by equation (1)

$$= U_{max}\left[1-\left(\frac{r}{R}\right)^2\right] = 1.5\left[1-\left(\frac{.06}{.1}\right)^2\right]$$

$$= 1.5[1.0 - .36] = 1.5 \times .64 = \mathbf{0.96 \text{ m/s. Ans.}}$$

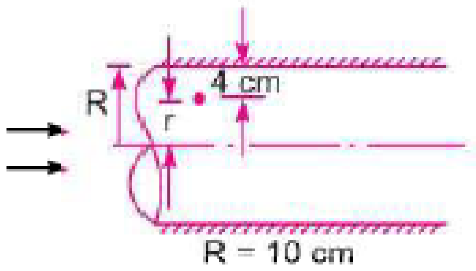
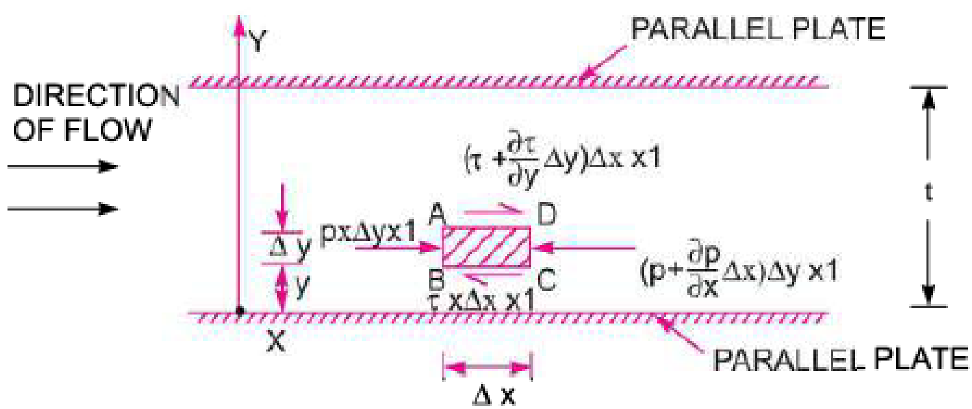


Fig. 9.4

- **Laminar Flow of fluid between two parallel plates:**
- Consider two parallel fixed plated at a distance 't' apart as shown in the Fig.
- A viscous fluid is flowing between these two plates from left to right.
- Consider a fluid element of length Δx and thickness Δy at a distance 'y' from the lower fixed plate and the width of the element as unity.
- If 'p' is the intensity of pressure on the face AB of the fluid element,
- Then intensity of pressure on the face CD will be

$$\left(p + \frac{\partial p}{\partial x}\Delta x\right).$$

- Let 'τ' be the shear stress acting on the face BC,
- then the shear stress on the face AD will be $\left(\tau + \frac{\partial \tau}{\partial y}\Delta y\right)$



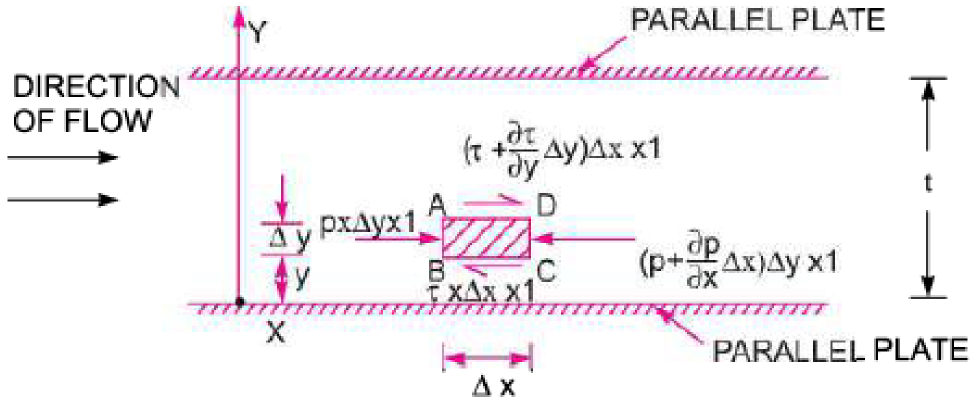
- **Laminar Flow of fluid between two parallel plates:**
- Then the forces acting on the fluid element are:

The pressure force, $p \times \Delta y \times 1$ on face AB.

The pressure force, $\left(p + \frac{\partial p}{\partial x}\Delta x\right)\Delta y \times 1$ on face CD.

The shear force, $\tau \times \Delta x \times 1$ on face BC.

The shear force, $\left(\tau + \frac{\partial \tau}{\partial y}\Delta y\right)\Delta x \times 1$ on face AD.



- **Laminar Flow of fluid between two parallel plates:**
- For steady and uniform flow, there is no acceleration and hence the resultant force in the direction of flow is zero.

$$p\Delta y \times 1 - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\Delta y \times 1 - \tau\Delta x \times 1 + \left(\tau + \frac{\partial \tau}{\partial y}\Delta y\right)\Delta x \times 1 = 0$$

$$-\frac{\partial p}{\partial x}\Delta x\Delta y + \frac{\partial \tau}{\partial x}\Delta y\Delta x = 0$$

Dividing by $\Delta x\Delta y$, we get $-\frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial y} = 0$ or $\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}$

- **Laminar Flow of fluid between two parallel plates:**
- (i) Velocity Distribution:
- To obtain the velocity distribution across a section, the value of shear stress from Newton's law of viscosity for laminar flow is substituted.

$$\tau = \mu \frac{du}{dy} \qquad \qquad \frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}$$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left(\mu \frac{du}{dy}\right) = \mu \frac{\partial^2 u}{dy^2}$$

$$\frac{\partial^2 u}{dy^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}$$

Integrating the above equation w.r.t. y, we get

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial p}{\partial x} y + C_1 \qquad \left\{\because \frac{\partial p}{\partial x} \text{ is constant}\right\}$$

Integrating again $$u = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{y^2}{2} + C_1 y + C_2$$

- **Laminar Flow of fluid between two parallel plates:**
- (i) Velocity Distribution:
- To obtain the velocity distribution across a section, the value of shear stress from Newton's law of viscosity for laminar flow is substituted.

where C_1 and C_2 are constants of integration. Their values are obtained from the two boundary conditions that is (*i*) at $y = 0$, $u = 0$ (*ii*) at $y = t$, $u = 0$.

$$u = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{y^2}{2} + C_1 y + C_2$$

- Now Substitute y = 0 and u = 0 in the above equation, we get

$$0 = 0 + C_1 \times 0 + C_2 \text{ or } C_2 = 0$$

- Again Substitute y = t and u = 0, we get;

$$0 = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{t^2}{2} + C_1 \times t + 0$$

$$C_1 = -\frac{1}{\mu}\frac{\partial p}{\partial x}\frac{t^2}{2 \times t} = -\frac{1}{2\mu}\frac{\partial p}{\partial x}t$$

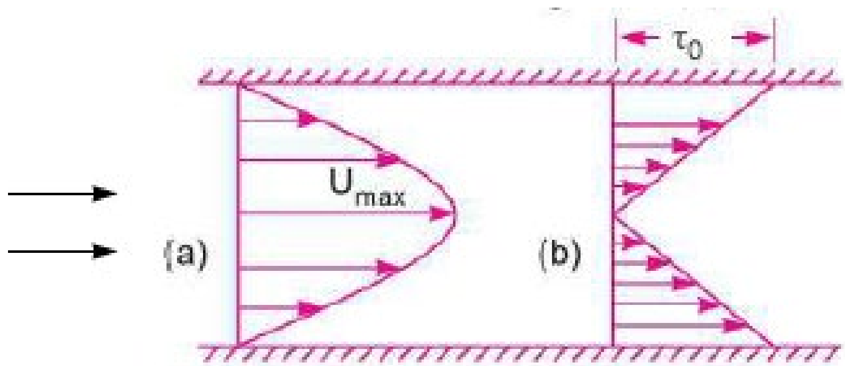
- **Laminar Flow of fluid between two parallel plates:**
- (i) Velocity Distribution:
- Now substitute the value of C1 and C2 in the below equation, we get;

$$u = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{y^2}{2} + C_1 y + C_2$$

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x} y^2 + y\left(-\frac{1}{2\mu}\frac{\partial p}{\partial x} t\right)$$

$$u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2]$$

- From the above equation, we see 'u' varies with the square of 'y' and the other terms are constant.
- Hence the above equation is a equation of PARABOLA and the velocity distribution across a section of a parallel plate is PARABOLIC.



- **Laminar Flow of fluid between two parallel plates:**
- (ii) Ratio of maximum velocity to average velocity:
- The velocity is maximum when y = t/2. Substitute this value in the equation, we get;

$$u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2]$$

$$U_{\max} = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[t \times \frac{t}{2} - \left(\frac{t}{2}\right)^2\right]$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{t^2}{2} - \frac{t^2}{4}\right] = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\frac{t^2}{4} = -\frac{1}{8\mu}\frac{\partial p}{\partial x}t^2$$

- **Laminar Flow of fluid between two parallel plates:**
- (ii) Ratio of maximum velocity to average velocity:

The average velocity, $\bar{u}$, is obtained by dividing the discharge (Q) across the section by the area of the section ($t \times 1$). And the discharge Q is obtained by considering the rate of flow of fluid through the strip of thickness dy and integrating it. The rate of flow through strip is

$$dQ = \text{Velocity at a distance } y \times \text{Area of strip}$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2] \times dy \times 1$$

$$Q = \int_0^t dQ = \int_0^t -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2]\, dy$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{ty^2}{2} - \frac{y^3}{3}\right]_0^t = \frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{t^3}{2} - \frac{t^3}{3}\right]$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\frac{t^3}{6} = -\frac{1}{12\mu}\frac{\partial p}{\partial x}t^3$$

$$\bar{u} = \frac{Q}{\text{Area}} = -\frac{\frac{1}{12\mu}\frac{\partial p}{\partial x} . t^3}{t \times 1} = -\frac{1}{12\mu}\frac{\partial p}{\partial x}t^2$$

- **Laminar Flow of fluid between two parallel plates:**
- (ii) Ratio of maximum velocity to average velocity:
- Divide the equation which we got for maximum and average velocity;

$$\frac{U_{max}}{\bar{u}} = \frac{-\frac{1}{8\mu}\frac{\partial p}{\partial x}t^2}{-\frac{1}{12\mu}\frac{\partial p}{\partial x}t^2} = \frac{12}{8} = \frac{3}{2}$$

- **Laminar Flow of fluid between two parallel plates:**
- (iii) Pressure drop for a given length:
- Average velocity

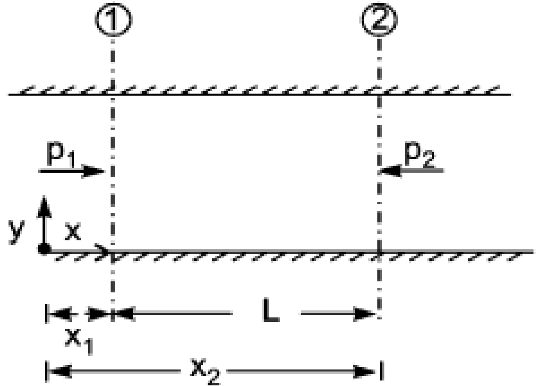
$$\bar{u} = -\frac{1}{12\mu}\frac{\partial p}{\partial x}t^2 \quad \text{or} \quad \frac{\partial p}{\partial x} = -\frac{12\mu\bar{u}}{t^2}$$

Integrating this equation w.r.t. x, we get

$$\int_2^1 dp = \int_2^1 -\frac{12\mu\bar{u}}{t^2}\,dx$$

$$p_1 - p_2 = -\frac{12\mu\bar{u}}{t^2}[x_1 - x_2] = \frac{12\mu\bar{u}}{t^2}[x_2 - x_1]$$

$$p_1 - p_2 = \frac{12\mu\bar{u}L}{t^2} \qquad [\because x_1 - x_2 = L]$$



If h_f is the drop of pressure head, then

$$h_f = \frac{p_1 - p_2}{\rho g} = \frac{12\mu\bar{u}L}{\rho g t^2}$$

- **Laminar Flow of fluid between two parallel plates:**
- (iv) Shear Stress distribution:
- It is obtained by substitute the value of u in Newton's law of viscosity; we get;

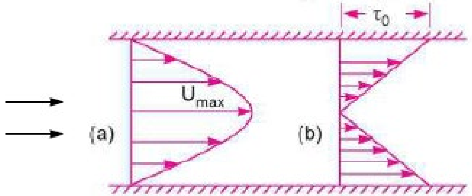
$$\tau = \mu \frac{\partial u}{\partial y} \qquad u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2]$$

$$\tau = \mu \frac{\partial u}{\partial y} = \mu \frac{\partial}{\partial y}\left[-\frac{1}{2\mu}\frac{\partial p}{\partial x}\left(ty - y^2\right)\right] = \mu\left[-\frac{1}{2\mu}\frac{\partial p}{\partial x}(t - 2y)\right]$$

$$\tau = -\frac{1}{2}\frac{\partial p}{\partial x}[t - 2y]$$

In equation (9.14), $\frac{\partial p}{\partial x}$ and t are constant. Hence τ varies linearly with y. The shear stress distribution is shown in Fig. 9.7 (b). Shear stress is maximum, when $y = 0$ or t at the walls of the plates. Shear stress is zero, when $y = t/2$ that is at the centre line between the two plates. Max. shear stress (τ_0) is given by

$$\tau_0 = -\frac{1}{2}\frac{\partial p}{\partial x}\,t.$$



- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**

- **<u>Reference:</u>**
- Fluid Mechanics by Fox
- Fluid Mechanics by Bansal
- Fluid Mechanics by Young
- NPTEL

CL24203 – FLUID MECHANICS

III Semester BTech (Chemical Engineering)

- **Module 3:**
- **Internal incompressible viscous flow:**
- **Introduction;**
- **flow of incompressible fluid in circular pipe;**
- **laminar flow for Newtonian fluid;**
- **Hagen-Poiseuille equation;**
- flow of Non-Newtonian fluid,
- introduction to turbulent flow in a pipe;
- energy consideration in pipe flow,
- **relation between average and maximum velocity,**
- Bernoulli’s equation–kinetic energy correction factor; head loss; friction factor; major and minor losses,
- Pipe fittings and valves. [8]

LECTURE PLAN AND LEARNING OBJECTIVES FOR 40 [ONE HOUR] LECTURES

Module 3: Internal incompressible viscous flow: Introduction; flow of incompressible fluid in circular pipe; laminar flow for Newtonian fluid; Hagen-Poiseuille equation; flow of Non-Newtonian fluid, introduction to turbulent flow in a pipe; energy consideration in pipe flow, relation between average and maximum velocity, Bernoulli's equation–kinetic energy correction factor; head loss; friction factor; major and minor losses, Pipe fittings and valves. [8]

Lecture I

Internal incompressible viscous flow: Introduction; flow of incompressible fluid in circular pipe;

Lecture II

Laminar flow for Newtonian fluid; Hagen-Poiseuille equation;

Lecture III

Flow of Non-Newtonian fluid,

Lecture IV

Introduction to turbulent flow in a pipe;

Lecture V

Energy consideration in pipe flow,

LECTURE PLAN AND LEARNING OBJECTIVES FOR 40 [ONE HOUR] LECTURES

Module 3: Internal incompressible viscous flow: Introduction; flow of incompressible fluid in circular pipe; laminar flow for Newtonian fluid; Hagen-Poiseuille equation; flow of Non-Newtonian fluid, introduction to turbulent flow in a pipe; energy consideration in pipe flow, relation between average and maximum velocity, Bernoulli's equation–kinetic energy correction factor; head loss; friction factor; major and minor losses, Pipe fittings and valves. [8]

Lecture VI

Relation between average and maximum velocity,

Lecture VII

Bernoulli's equation–kinetic energy correction factor;

Lecture VIII

Head loss; friction factor; major and minor losses, Pipe fittings and valves.

- **Equation of Motion:**
- A dynamic equation describing fluid motion may be obtained by applying Newton's second law to a particle.
- To derive the differential form of the momentum equation, we shall apply Newton's second law to an infinitesimal fluid particle of mass dm.

Newton's second law for a finite system is given by

$$\vec{F} = \left.\frac{d\vec{P}}{dt}\right)_{\text{system}}$$

$\vec{F}$ is the **net external force** acting on the system,
$\vec{P}$ is the **linear momentum** of the system, and
$\frac{d\vec{P}}{dt}$ is the **rate of change of momentum** of the system.

- This equation states that the **time rate of change of momentum of a system equals the net external force** acting on it.
- **Linear momentum** is a fundamental concept in mechanics that describes the motion of a body.

$$\vec{P} = m\vec{V}$$

$\vec{P}$ = linear momentum (vector quantity)
m = mass of the body (scalar)
$\vec{V}$ = velocity of the body (vector)

- **Equation of Motion:**
- A dynamic equation describing fluid motion may be obtained by applying Newton's second law to a particle.
- To derive the differential form of the momentum equation, we shall apply Newton's second law to an infinitesimal fluid particle of mass dm.

Newton's second law for a finite system is given by

$$\vec{F} = \left.\frac{d\vec{P}}{dt}\right)_{\text{system}}$$

where the linear momentum, $\vec{P}$, of the system is given by

$$\vec{P}_{\text{system}} = \int_{\text{mass (system)}} \vec{V}\, dm$$

Then, for an infinitesimal system of mass dm, Newton's second law can be written

$$d\vec{F} = dm \left.\frac{d\vec{V}}{dt}\right)_{\text{system}}$$

- **Equation of Motion:**
- We can write the Newton's second law as the vector equation: equation shown is an **expanded form of Newton's second law** in **differential form**, using the **material derivative.**

$$d\vec{F} = dm\frac{D\bar{V}}{Dt} = dm\left[u\frac{\partial \bar{V}}{\partial x} + v\frac{\partial \bar{V}}{\partial y} + w\frac{\partial \bar{V}}{\partial z} + \frac{\partial \bar{V}}{\partial t}\right]$$

The operator

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

represents the rate of change following a moving fluid particle.

Local acceleration ($\partial\vec{V}/\partial t$) → unsteady effects

Convective acceleration ($u\partial\vec{V}/\partial x + v\partial\vec{V}/\partial y + w\partial\vec{V}/\partial z$) → due to spatial velocity changes.

- This expression represents the **force per fluid element** due to the **acceleration of that element** as it moves through a velocity field that varies in both space and time.
- In general, two types of forces need to be considered: **surface forces,** which act on the surface of the differential element, and **body forces,** which are distributed throughout the element.

- **Equation of Motion:**
- We can write the Newton's second law as the vector equation: equation shown is an **expanded form of Newton's second law** in **differential form**, using the **material derivative.**

$$d\bar{F} = dm\frac{D\bar{V}}{Dt} = dm\left[u\frac{\partial \bar{V}}{\partial x} + v\frac{\partial \bar{V}}{\partial y} + w\frac{\partial \bar{V}}{\partial z} + \frac{\partial \bar{V}}{\partial t}\right]$$

- In general, two types of forces need to be considered: **surface forces,** which act on the surface of the differential element, and **body forces,** which are distributed throughout the element.
- Body force, Fb, of interest is the weight of the element, which can be expressed as

$$\delta \mathbf{F}_b = \delta m\, \mathbf{g}$$

where $\mathbf{g}$ is the vector representation of the acceleration of gravity. In component form

$$\delta F_{bx} = \delta m\, g_x$$

$$\delta F_{by} = \delta m\, g_y$$

$$\delta F_{bz} = \delta m\, g_z$$

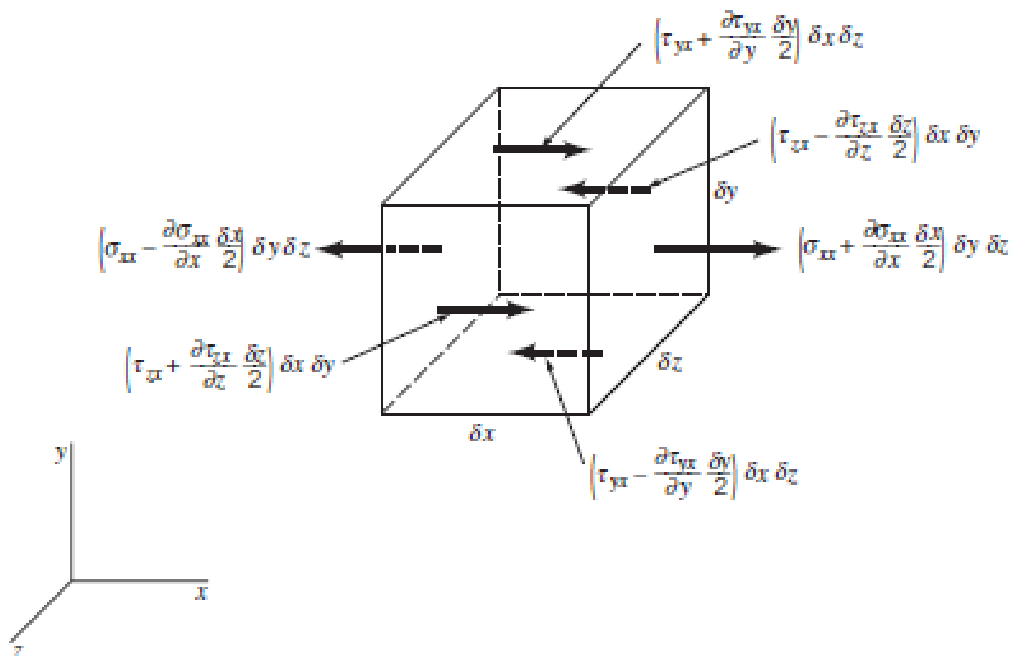
where g_x, g_y, and g_z are the components of the acceleration of gravity vector in the x, y, and z directions, respectively.

- **Equation of Motion:**
- Surface forces act on the element as a result of its interaction with its surroundings.
- Normal Stress is

$$\sigma_n = \lim_{\delta A \to 0} \frac{\delta F_n}{\delta A}$$

and the *shearing stresses* are defined as

$$\tau_1 = \lim_{\delta A \to 0} \frac{\delta F_1}{\delta A}$$



- **Equation of Motion:**
- We shall consider the x component of the force acting on a differential element of mass dm and volume dV = dx dy dz.
- Only those stresses that act in the x direction will give rise to surface forces in the x direction.

To obtain the net surface force in the x direction, dF_{S_x}, we must sum the forces in the x direction. Thus,

$$\begin{aligned}dF_{S_x} = &\left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}\right) dy\,dz - \left(\sigma_{xx} - \frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}\right) dy\,dz \\ &+ \left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\frac{dy}{2}\right) dx\,dz - \left(\tau_{yx} - \frac{\partial \tau_{yx}}{\partial y}\frac{dy}{2}\right) dx\,dz \\ &- \left(\tau_{zx} - \frac{\partial \tau_{zx}}{\partial z}\frac{dz}{2}\right) dx\,dy - \left(\tau_{zx} - \frac{\partial \tau_{zx}}{\partial z}\frac{dz}{2}\right) dx\,dy\end{aligned}$$

On simplifying, we obtain

$$dF_{S_x} = \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) dx\,dy\,dz$$

- **Equation of Motion:**
- We shall consider the x component of the force acting on a differential element of mass dm and volume dV = dx dy dz.
- Only those stresses that act in the x direction will give rise to surface forces in the x direction.

When the force of gravity is the only body force acting, then the body force per unit mass is $\vec{g}$. The net force in the x direction, dF_x, is given by

$$dF_x = dF_{B_x} + dF_{S_x} = \left(\rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) dx\, dy\, dz \qquad (5.23a)$$

We can derive similar expressions for the force components in the y and z directions:

$$dF_y = dF_{B_y} + dF_{S_y} = \left(\rho g_y + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \right) dx\, dy\, dz \qquad (5.23b)$$

$$dF_z = dF_{B_z} + dF_{S_z} = \left(\rho g_z + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} \right) dx\, dy\, dz \qquad (5.23c)$$

- **Equation of Motion:**
- We shall consider the x component of the force acting on a differential element of mass dm and volume dV = dx dy dz.

$$\delta F_x = \delta m\, a_x$$

$$\delta \mathbf{F} = \delta \mathbf{F}_s + \delta \mathbf{F}_b.$$

$$\delta F_y = \delta m\, a_y$$

$$\delta F_z = \delta m\, a_z$$

- Now substitute the values, We get:

$$\delta m = \rho\, \delta x\, \delta y\, \delta z,$$

$$\rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

$$\rho g_y + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)$$

$$\rho g_z + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)$$

- The above Equations are the general differential equations of motion for a fluid.

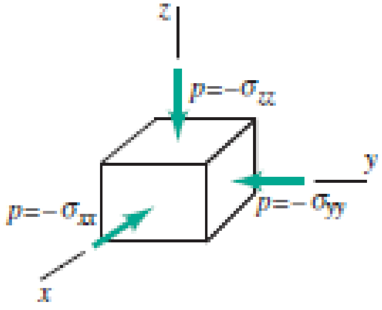
- **Equation of Motion:**
- Flow fields in which the shearing stresses are assumed to be negligible are said to be inviscid.
- As discussed in the previous lectures, for fluids in which there are no shearing stresses, the normal stress at a point is independent of direction—that is,

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz}.$$

- In this instance we define the pressure, p, as the negative of the normal stress so that, as indicated by the figure in the margin,

$$-p = \sigma_{xx} = \sigma_{yy} = \sigma_{zz}$$

- The negative sign is used so that a compressive normal stress (which is what we expect in a fluid) will give a positive value for p.



- **Equation of Motion:**
- For an inviscid flow in which all the shearing stresses are zero, and the normal stresses are replaced by -p, the general equations of motion reduce to:

$$\rho g_x - \frac{\partial p}{\partial x} = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

$$\rho g_y - \frac{\partial p}{\partial y} = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)$$

$$\rho g_z - \frac{\partial p}{\partial z} = \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)$$

- Or in vector form

$$\rho\frac{D\vec{V}}{Dt} = \rho\vec{g} - \nabla p \qquad \rho\mathbf{g} - \nabla p = \rho\left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V}\right]$$

- The above equation are commonly referred to as Euler’s equation of motion.

- **Equation of Motion:**
- Remember Continuity Equation

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} + \frac{\partial \rho}{\partial t} = 0$$

Since the vector operator, ∇, in rectangular coordinates, is given by

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = \nabla \cdot \rho \vec{V}$$

- We can rewrite the continuity equation in vector form as:

$$\nabla \cdot \rho \vec{V} + \frac{\partial \rho}{\partial t} = 0$$

- **Equation of Motion:**
- For incompressible fluids, density is constant (ρ = Constant)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \nabla \cdot \bar{V} = 0 \qquad \bar{V}(x, y, z, t),$$

For *steady* flow, all fluid properties are, by definition, independent of time. Thus $\partial\rho/\partial t = 0$ and at most $\rho = \rho(x, y, z)$. For steady flow, the continuity equation can be written as

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = \nabla \cdot \rho\bar{V} = 0 \tag{5.1d}$$

- **Equation of Motion:**
- Acceleration field

$$\frac{D\vec{V}}{Dt} \equiv \vec{a}_p = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} + \frac{\partial \vec{V}}{\partial t}$$

- Convective acceleration in vector form:

$$u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} = (\vec{V}\cdot\nabla)\vec{V}$$

- Now the acceleration field represented in vector form as:

$$\frac{D\vec{V}}{Dt} \equiv \vec{a}_p = (\vec{V}\cdot\nabla)\vec{V} + \frac{\partial \vec{V}}{\partial t}$$

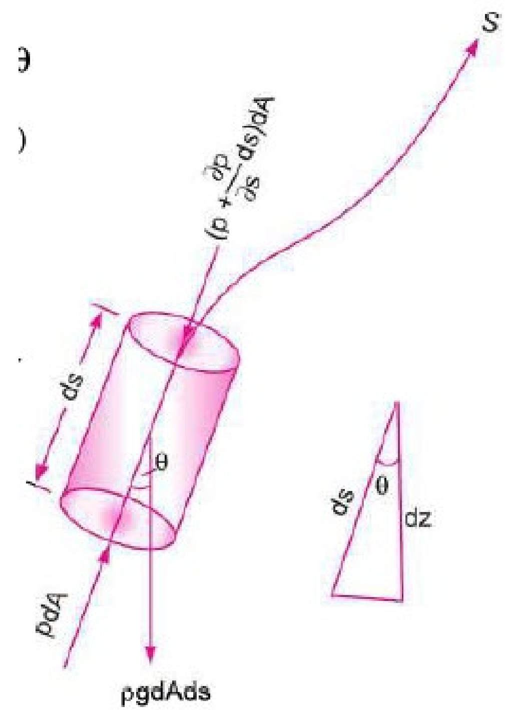
- **Euler's Equation in Streamline Coordinates:**
- Consider a stream-line in which flow is taking place in s- direction as shown in the Fig.
- Consider a cylindrical element of cross-section dA and length ds.
- The forces acting on the cylindrical element are:

Pressure force pdA in the direction of flow.

Pressure force $\left(p + \frac{\partial p}{\partial s} ds\right) dA$ opposite to the direction of flow.

Weight of element $\rho g dA ds$.

- Let θ is the angle between the direction of flow and the
- Line of action of the weight of element.



- **Euler's Equation in Streamline Coordinates:**
- The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element x acceleration in the direction of s.
- F = mass x acceleration.

$$pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g dA ds \cos\theta = \rho dA ds \times a_s$$

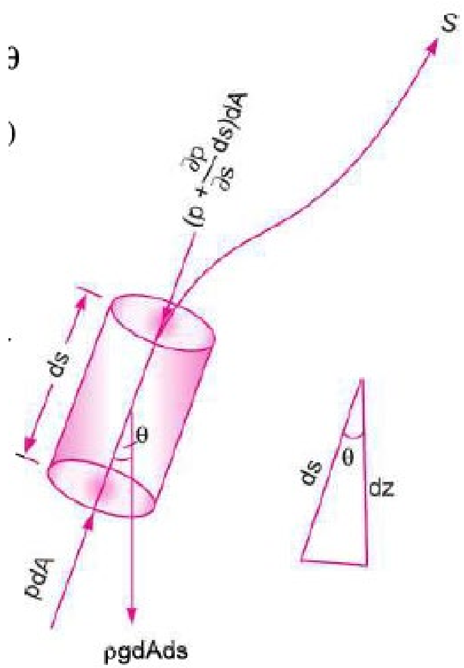
- Where a_s is the acceleration in the direction of s. (using Chain Rule of Differentiation)

$$a_s = \frac{dv}{dt}, \text{ where } v \text{ is a function of } s \text{ and } t.$$

$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\partial v}{\partial s} + \frac{\partial v}{\partial t} \quad \left\{\because \frac{ds}{dt} = v\right\}$$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$

$$\therefore \qquad a_s = \frac{v\partial v}{\partial s}$$



- **Euler's Equation in Streamline Coordinates:**
- Now substitute the value of a_s and simplify the equation, we get:

$$-\frac{\partial p}{\partial s}\, dsdA - \rho g\, dAds \cos\theta = \rho dAds \times \frac{\partial v}{\partial s}$$

Dividing by $\rho dsdA$, $-\frac{\partial p}{\rho \partial s} - g\cos\theta = \frac{v\partial v}{\partial s}$

or $$\frac{\partial p}{\rho \partial s} + g\cos\theta + v\frac{\partial v}{\partial s} = 0$$

$$\frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} + \frac{vdv}{ds} = 0 \quad \text{or} \quad \frac{dp}{\rho} + gdz + vdv = 0$$ we have $\cos\theta = \frac{dz}{ds}$

$$\frac{dp}{\rho} + gdz + vdv = 0$$

- The above equation is the Euler's equation of motion in streamline corordinates.

- **Bernoulli's Equation from Euler's Equation:**
- Now substitute the value of a_s and simplify the equation, we get:

Bernoulli's equation is obtained by integrating the Euler's equation of motion (

$$\int \frac{dp}{\rho} + \int g dz + \int v dv = \text{constant}$$

If flow is incompressible, ρ is constant and

$$\therefore \qquad \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

$$\text{or} \qquad \frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

$$\text{or} \qquad \frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

- The above equation are the Bernoulli's equation

- **Bernoulli's Equation from Euler's Equation:**
- Now substitute the value of a_s and simplify the equation, we get:

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or pressure head.

$v^2/2g$ = kinetic energy per unit weight or kinetic head.

z = potential energy per unit weight or potential head.

The following are the assumptions made in the derivation of Bernoulli's equation :

(*i*) The fluid is ideal, *i.e.*, viscosity is zero

(*ii*) The flow is steady

(*iii*) The flow is incompressible

(*iv*) The flow is irrotational.

- **Navier Stokes Equation:**
- For incompressible, Newtonian fluids it is known that the stresses are linearly related to the rates of deformation and can be expressed in Cartesian coordinates as (for normal stresses)

$$\sigma_{xx} = -p + 2\mu \frac{\partial u}{\partial x}$$

$$\sigma_{yy} = -p + 2\mu \frac{\partial v}{\partial y}$$

$$\sigma_{zz} = -p + 2\mu \frac{\partial w}{\partial z}$$

(for shearing stresses)

$$\tau_{xy} = \tau_{yx} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

$$\tau_{yz} = \tau_{zy} = \mu \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)$$

$$\tau_{zx} = \tau_{xz} = \mu \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)$$

- **Navier Stokes Equation:**
- For incompressible, Newtonian fluids it is known that the stresses are linearly related to the rates of deformation and can be expressed in Cartesian coordinates as (for normal stresses)

In any continuum (solid or fluid), the **stress tensor $\boldsymbol{\sigma}$** represents internal forces per unit area.

For a **fluid**, stress arises from two sources:

1. **Isotropic pressure** (acts equally in all directions)
2. **Viscous stresses** (caused by velocity gradients → internal friction between adjacent fluid layers)

So, we can always decompose:

$$\boxed{\boldsymbol{\sigma} = -p\,\mathbf{I} + \boldsymbol{\tau}}$$

where

- p = thermodynamic pressure
- $\mathbf{I}$ = identity tensor
- $\boldsymbol{\tau}$ = **deviatoric (viscous) stress tensor**, caused by deformation (velocity gradients)

- **Navier Stokes Equation:**
- For incompressible, Newtonian fluids it is known that the stresses are linearly related to the rates of deformation and can be expressed in Cartesian coordinates as (for normal stresses)
- Newton's law of viscosity (generalized form)

Newton postulated that the **viscous stresses are linearly proportional** to the **rates of deformation (strain rates)** in the fluid.

For a Newtonian fluid:

$$\boxed{\tau_{ij} = 2\mu\, e_{ij} + \lambda\,(\nabla \cdot \mathbf{v})\,\delta_{ij}}$$

where

- μ = **dynamic viscosity** (resistance to shear)
- λ = **second coefficient of viscosity (bulk viscosity)**
- $e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$ = rate-of-strain tensor
- $\nabla \cdot \mathbf{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$ = volumetric strain rate

- **Navier Stokes Equation:**
- For incompressible, Newtonian fluids it is known that the stresses are linearly related to the rates of deformation and can be expressed in Cartesian coordinates as (for normal stresses)

For $i = j$, say xx-component:

$$\tau_{xx} = 2\mu e_{xx} + \lambda(\nabla \cdot \mathbf{v})\delta_{xx}$$

Since $\delta_{xx} = 1$ and $e_{xx} = \frac{\partial u}{\partial x}$,

$$\tau_{xx} = 2\mu \frac{\partial u}{\partial x} + \lambda(\nabla \cdot \mathbf{v})$$

Thus, total **normal stress** on an x-plane:

$$\sigma_{xx} = -p + \tau_{xx} = -p + 2\mu \frac{\partial u}{\partial x} + \lambda(\nabla \cdot \mathbf{v})$$

$$\sigma_{yy} = -p + 2\mu \frac{\partial v}{\partial y} + \lambda(\nabla \cdot \mathbf{v})$$

$$\sigma_{zz} = -p + 2\mu \frac{\partial w}{\partial z} + \lambda(\nabla \cdot \mathbf{v})$$

- **Navier Stokes Equation:**
- For incompressible, Newtonian fluids it is known that the stresses are linearly related to the rates of deformation and can be expressed in Cartesian coordinates as (for normal stresses)

For **incompressible** fluids,

$$\nabla \cdot \mathbf{v} = 0,$$

so the λ-term drops out, giving:

$$\boxed{\sigma_{xx} = -p + 2\mu \frac{\partial u}{\partial x}, \quad \sigma_{yy} = -p + 2\mu \frac{\partial v}{\partial y}, \quad \sigma_{zz} = -p + 2\mu \frac{\partial w}{\partial z}.}$$

- **Navier Stokes Equation:**
- For incompressible, Newtonian fluids it is known that the stresses are linearly related to the rates of deformation and can be expressed in Cartesian coordinates as (for normal stresses)

For a Newtonian fluid the deviatoric (viscous) stress is proportional to the rate-of-strain:

$$\tau_{ij} = 2\mu\, e_{ij} + \lambda(\nabla\cdot\mathbf{v})\,\delta_{ij}, \qquad e_{ij} = \tfrac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right).$$

For **off-diagonal** components $i \neq j$ the Kronecker delta $\delta_{ij} = 0$, so the bulk-viscosity term drops out and

$$\tau_{ij} = 2\mu\, e_{ij} = \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right).$$

Writing these with usual velocity notation $(u, v, w) = (v_1, v_2, v_3)$:

$$\boxed{\begin{aligned} \tau_{xy} &= \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \\ \tau_{xz} &= \tau_{zx} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right), \\ \tau_{yz} &= \tau_{zy} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right). \end{aligned}}$$

- **Navier Stokes Equation:**
- Now the stresses can be substituted in the differential equation of motion and simplify by using the continuity equation for incompressible flow.

$$\rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right)$$

$$\rho g_y + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} = \rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right)$$

$$\rho g_z + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = \rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right)$$

- **Navier Stokes Equation:**
- Now the stresses can be substituted in the differential equation of motion and simplify by using the continuity equation for incompressible flow.

(x direction)

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

(y direction)

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\rho g_y+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

(z direction)

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\rho g_z+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

- These equations are commonly called the famous **Navier–Stokes equations**

- **Navier Stokes Equation:**
- Now we can if for frictionless force ($\mu = 0$). The above equation will be reduced to Euler's equation of motion.

$$\rho g_x - \frac{\partial p}{\partial x} = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

$$\rho g_y - \frac{\partial p}{\partial y} = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)$$

$$\rho g_z - \frac{\partial p}{\partial z} = \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)$$

- **Bernoulli's Equation for Real Fluids:**
- Bernoulli's equation was derived on the assumption that fluid is inviscid and therefore frictionless.
- But all the real fluids are viscous and hence offer resistance to flow.
- Thus there are always some losses in fluid and hence in the application of Bernoulli's equation, these losses have to be taken into consideration.
- Thus the Bernoulli's equation for real fluids between points 1 and 2 is given as:

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$$

where h_L is loss of energy between points 1 and 2.

- **Bernoulli's Equation for Real Fluids:**

Problem 6.7 *A pipe of diameter 400 mm carries water at a velocity of 25 m/s. The pressures at the points A and B are given as 29.43 N/cm² and 22.563 N/cm² respectively while the datum head at A and B are 28 m and 30 m. Find the loss of head between A and B.*

Solution. Given :

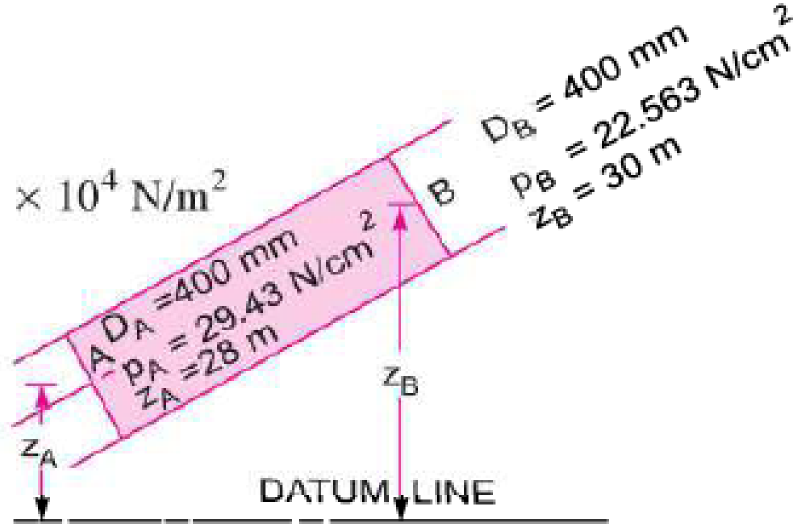
Dia. of pipe, $D = 400 \text{ mm} = 0.4 \text{ m}$

Velocity, $V = 25 \text{ m/s}$

At point A, $p_A = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

$z_A = 28 \text{ m}$

$v_A = v = 25 \text{ m/s}$



$\therefore$ Total energy at A, $$E_A = \frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A$$

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 28$$

$$= 30 + 31.85 + 28 = 89.85 \text{ m}$$

- **Bernoulli's Equation for Real Fluids:**

Problem 6.7 *A pipe of diameter 400 mm carries water at a velocity of 25 m/s. The pressures at the points A and B are given as 29.43 N/cm² and 22.563 N/cm² respectively while the datum head at A and B are 28 m and 30 m. Find the loss of head between A and B.*

At point *B*, $p_B = 22.563 \text{ N/cm}^2 = 22.563 \times 10^4 \text{ N/m}^2$

$z_B = 30 \text{ m}$

$v_B = v = v_A = 25 \text{ m/s}$

$\therefore$ Total energy at B, $E_B = \frac{p_B}{\rho g} + \frac{v_B^2}{2g} + z_B$

$$= \frac{22.563 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 30 = 23 + 31.85 + 30 = 84.85 \text{ m}$$

$\therefore$ Loss of energy $= E_A - E_B = 89.85 - 84.85 =$ **5.0 m. Ans.**

- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**

- **<u>Reference:</u>**
- Fluid Mechanics by Fox
- Fluid Mechanics by Bansal
- Fluid Mechanics by Young
- NPTEL

CL24203 – FLUID MECHANICS

III Semester BTech (Chemical Engineering)

CL203 FLUID MECHANICS

- **Module 3:**
- **Internal incompressible viscous flow:**
- **Introduction;**
- **flow of incompressible fluid in circular pipe;**
- **laminar flow for Newtonian fluid;**
- **Hagen-Poiseuille equation;**
- flow of Non-Newtonian fluid,
- introduction to turbulent flow in a pipe;
- **energy consideration in pipe flow,**
- **relation between average and maximum velocity,**
- **Bernoulli’s equation–kinetic energy correction factor**; head loss; friction factor; major and minor losses,
- Pipe fittings and valves. [8]

LECTURE PLAN AND LEARNING OBJECTIVES FOR 40 [ONE HOUR] LECTURES

Module 3: Internal incompressible viscous flow: Introduction; flow of incompressible fluid in circular pipe; laminar flow for Newtonian fluid; Hagen-Poiseuille equation; flow of Non-Newtonian fluid, introduction to turbulent flow in a pipe; energy consideration in pipe flow, relation between average and maximum velocity, Bernoulli's equation–kinetic energy correction factor; head loss; friction factor; major and minor losses, Pipe fittings and valves. [8]

Lecture I

Internal incompressible viscous flow: Introduction; flow of incompressible fluid in circular pipe;

Lecture II

Laminar flow for Newtonian fluid; Hagen-Poiseuille equation;

Lecture III

Flow of Non-Newtonian fluid,

Lecture IV

Introduction to turbulent flow in a pipe;

Lecture V

Energy consideration in pipe flow,

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Module 3: Internal incompressible viscous flow: Introduction; flow of incompressible fluid in circular pipe; laminar flow for Newtonian fluid; Hagen-Poiseuille equation; flow of Non-Newtonian fluid, introduction to turbulent flow in a pipe; energy consideration in pipe flow, relation between average and maximum velocity, Bernoulli's equation–kinetic energy correction factor; head loss; friction factor; major and minor losses, Pipe fittings and valves. [8]

Lecture VI

Relation between average and maximum velocity,

Lecture VII

Bernoulli's equation–kinetic energy correction factor;

Lecture VIII

Head loss; friction factor; major and minor losses, Pipe fittings and valves.

- **First Law of Thermodynamics:**
- The first law of thermodynamics is a statement of conservation of energy for a system:

$$\delta Q - \delta W = dE$$

- Q = heat added to the system (positive if added, negative if removed) and W is the Work done by the system (positive if done by the system, negative if done on the system) and dE = Change in internal energy of the system
- The equation can be written in rate form as

$$\dot{Q} - \dot{W} = \left.\frac{dE}{dt}\right)_{\text{system}}$$

- where the total energy of the system is given by

$$E_{\text{system}} = \int_{M(\text{system})} e\, dm = \int_{\forall(\text{system})} e\, \rho\, d\forall$$

- **First Law of Thermodynamics:**
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- The equation can be written in rate form as

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- where the total energy of the system is given by

$$E_{\text{system}} = \int_{M(\text{system})} e\, dm = \int_{V\!\!\!\!-(\text{system})} e\, \rho\, dV\!\!\!\!-$$

- The system energy per unit mass e may be of several types:

$$e = e_{\text{internal}} + e_{\text{kinetic}} + e_{\text{potential}} + e_{\text{other}}$$

- **First Law of Thermodynamics:**
- The system energy per unit mass e may be of several types:

$$e = e_{\text{internal}} + e_{\text{kinetic}} + e_{\text{potential}} + e_{\text{other}}$$

where e_{other} could encompass chemical reactions, nuclear reactions, and electrostatic or magnetic field effects. We neglect e_{other} here and consider only the first three terms

$$e = \hat{u} + \frac{1}{2}V^2 + gz$$

- Where u is the specific internal energy, V the speed, and z the height (relative to a convenient datum) of a particle of substance having mass dm.
- To derive the control volume formulation of the first law of thermodynamics, we set
- N = E and η = e in RTT.

$$\left.\frac{dN}{dt}\right)_{\text{system}} = \frac{\partial}{\partial t}\int_{\text{CV}} \eta\,\rho\,dV\!\!\!\!- + \int_{\text{CS}} \eta\,\rho\vec{V}\cdot d\vec{A}$$

$$\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \rho_2 A_2 V_2 b_2 - \rho_1 A_1 V_1 b_1$$

$$\left.\frac{dE}{dt}\right)_{\text{system}} = \frac{\partial}{\partial t}\int_{\text{CV}} e\,\rho\,dV\!\!\!\!- + \int_{\text{CS}} e\,\rho\vec{V}\cdot d\vec{A}$$

- **First Law of Thermodynamics:**
- Since the system and the control volume coincide at t_0

$$\left[\dot{Q} - \dot{W}\right]_{\text{system}} = \left[\dot{Q} - \dot{W}\right]_{\text{control volume}}$$

- We can write the equation as:

$$\dot{Q} - \dot{W} = \frac{\partial}{\partial t}\int_{\text{CV}} e\,\rho\,d\forall + \int_{\text{CS}} e\,\rho\vec{V}\cdot d\vec{A}$$

- Where e is:

$$e = \hat{u} + \tfrac{1}{2}V^2 + gz$$

- The above equation is the control volume formula for the first law of thermodynamics:

- **First Law of Thermodynamics:**
- The term Ŵ in the above equation has a positive numerical value when work is done by the control volume on the surroundings.
- The rate of work done on the control volume is of opposite sign to the work done by the control volume.
- The rate of work done by the control volume is conveniently subdivided into four classifications,

$$\dot{W} = \dot{W}_s + \dot{W}_{\text{normal}} + \dot{W}_{\text{shear}} + \dot{W}_{\text{other}}$$

- **SHAFT WORK:**
- We shall designate shaft work Ŵs and hence the rate of work transferred out through the control surface by shaft work is designated Ŵs.
- Examples of shaft work are the work produced by the steam turbine (positive shaft work) of a power plant, and the work input required to run the compressor of a refrigerator (negative shaft work).

- **First Law of Thermodynamics:**
- **Work done by normal stresses at the control surface:**
- Recall that work requires a force to act through a distance.
- Thus, when a force, F, acts through an infinitesimal displacement, ds, the work done is given by

$$\delta W = \vec{F} \cdot d\vec{s}$$

- To obtain the rate at which work is done by the force, divide by the time increment, Δt

$$\dot{W} = \lim_{\Delta t \to 0} \frac{\delta W}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{F} \cdot d\vec{s}}{\Delta t} \quad \text{or} \quad \dot{W} = \vec{F} \cdot \vec{V}$$

- Hence the rate of work done on the area element is   Normal Stress = Force/Area

$$d\vec{F}_{\text{normal}} \cdot \vec{V} = \sigma_{nn}\, d\vec{A} \cdot \vec{V}$$

- **First Law of Thermodynamics:**
- **Work done by normal stresses at the control surface:**
- Since the work out across the boundaries of the control volume is the negative of the work done on the control volume,
- the total rate of work out of the control volume due to normal stresses is

$$\dot{W}_{\text{normal}} = -\int_{CS} \sigma_{nn}\, d\vec{A} \cdot \vec{V} = -\int_{CS} \sigma_{nn} \vec{V} \cdot d\vec{A}$$

- **Work done by Shear stresses at the control surface:**
- Just as work is done by the normal stresses at the boundaries of the control volume, so may work be done by the shear stresses.

$$d\vec{F}_{\text{shear}} = \vec{\tau}\, dA$$

$$\int_{CS} \vec{\tau}\, dA \cdot \vec{V} = \int_{CS} \vec{\tau} \cdot \vec{V}\, dA$$

- **First Law of Thermodynamics:**
- **Work done by Shear stresses at the control surface:**
- Since the work out across the boundaries of the control volume is the negative of the work done on the control volume,
- the rate of work out of the control volume; due to shear stresses is given by

$$\dot{W}_{shear} = -\int_{CS} \vec{\tau} \cdot \vec{V} \, dA$$

- This integral is better expressed as three terms

$$\begin{aligned}\dot{W}_{shear} &= -\int_{CS} \vec{\tau} \cdot \vec{V} \, dA \\ &= -\int_{A(\text{shafts})} \vec{\tau} \cdot \vec{V} \, dA - \int_{A(\text{solid surface})} \vec{\tau} \cdot \vec{V} \, dA - \int_{A(\text{ports})} \vec{\tau} \cdot \vec{V} \, dA\end{aligned}$$

We have already accounted for the first term, since we included $\dot{W}_s$ previously. At solid surfaces, $\vec{V} = 0$, so the second term is zero (for a fixed control volume). Thus,

$$\dot{W}_{shear} = -\int_{A(\text{ports})} \vec{\tau} \cdot \vec{V} \, dA$$

- **First Law of Thermodynamics:**
- **Work done by Shear stresses at the control surface:**

This last term can be made zero by proper choice of control surfaces. If we choose a control surface that cuts across each port perpendicular to the flow, then $d\vec{A}$ is parallel to $\vec{V}$. Since $\vec{\tau}$ is in the plane of dA, $\vec{\tau}$ is perpendicular to $\vec{V}$. Thus, for a control surface perpendicular to $\vec{V}$,

$$\vec{\tau} \cdot \vec{V} = 0 \qquad \text{and} \qquad \dot{W}_{\text{shear}} = 0$$

- **Choose control surfaces along streamlines** of the fluid: No tangential velocity relative to the surface → shear stress does no work.
- **Avoid including solid walls in control surfaces**:
- **Choose boundaries where the fluid moves parallel to the wall at zero relative velocity**: Shear stress τ acts tangentially but $\vec{v}_{relative} = 0 \rightarrow \tau \cdot v = 0$.

- **First Law of Thermodynamics:**

- **Other Work:**
- Electrical energy could be added to the control volume. Also electromagnetic energy] e.g., in radar or laser beams, could be absorbed. In most problems, such contributions] will be absent, but we should note them in our general formulation.

With all of the terms in $\dot{W}$ evaluated, we obtain

$$\dot{W} = \dot{W}_s - \int_{CS} \sigma_{nn} \vec{V} \cdot d\vec{A} + \dot{W}_{shear} + \dot{W}_{other}$$

- **First Law of Thermodynamics:**
- **Control Volume Equation:**
- Substitute the expression of rate of work done in first law, we get:

$$\dot{Q} - \dot{W}_s + \int_{CS} \sigma_{nn} \vec{V} \cdot d\vec{A} - \dot{W}_{\text{shear}} - \dot{W}_{\text{other}} = \frac{\partial}{\partial t} \int_{CV} e \, \rho \, d\forall + \int_{CS} e \, \rho \vec{V} \cdot d\vec{A}$$

- Rearrange the terms:

$$\dot{Q} - \dot{W}_s - \dot{W}_{\text{shear}} - \dot{W}_{\text{other}} = \frac{\partial}{\partial t} \int_{CV} e \, \rho \, d\forall + \int_{CS} e \, \rho \vec{V} \cdot d\vec{A} - \int_{CS} \sigma_{nn} \vec{V} \cdot d\vec{A}$$

Since $\rho = 1/v$, where v is *specific volume*, then

$$\int_{CS} \sigma_{nn} \vec{V} \cdot d\vec{A} = \int_{CS} \sigma_{nn} \, v \, \rho \vec{V} \cdot d\vec{A}$$

$$\dot{Q} - \dot{W}_s - \dot{W}_{\text{shear}} - \dot{W}_{\text{other}} = \frac{\partial}{\partial t} \int_{CV} e \, \rho \, d\forall + \int_{CS} (e - \sigma_{nn} v) \, \rho \vec{V} \cdot d\vec{A}$$

- **First Law of Thermodynamics:**
- **Control Volume Equation:**
- We know that the normal stress is negative of the thermodynamic pressure –p. Hence $\sigma_{nn} = -p.$

$$\dot{Q} - \dot{W}_s - \dot{W}_{\text{shear}} - \dot{W}_{\text{other}} = \frac{\partial}{\partial t}\int_{CV} e\,\rho\,dV + \int_{CS} (e + pv)\,\rho\vec{V}\cdot d\vec{A}$$

- Now substitute the value of e in the last term we get the familiar energy equation of first law for a control volume as:

$$\dot{Q} - \dot{W}_s - \dot{W}_{\text{shear}} - \dot{W}_{\text{other}} = \frac{\partial}{\partial t}\int_{CV} e\,\rho\,dV + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A}$$

- Each work term in the above equation represents the rate of work done by the control volume on the surroundings.
- Note that in thermodynamics, for convenience, the combination u + pv (the fluid internal energy plus what is often called the "flow work") is usually replaced with enthalpy, h = u + pv (this is one of the reasons h was invented).

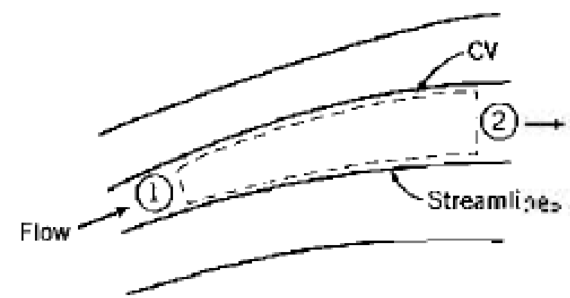
- **Bernoulli's Equation interpreted as an Energy Equation:**
- An equation identical in form to Bernoulli's equation (although requiring very different restrictions) may be obtained from the first law of thermodynamics.
- Our objective in this section is to reduce the energy equation to the form of the Bernoulli equation.
- Consider steady flow in the absence of shear forces. We choose a control volume bounded by streamlines along its periphery. Such a boundary, shown in Fig. often is called a stream tube.

Basic equation:

$$\dot{Q} - \underbrace{\cancel{\dot{W}_s}}_{=0(1)} - \underbrace{\cancel{\dot{W}_{\text{shear}}}}_{=0(2)} - \underbrace{\cancel{\dot{W}_{\text{other}}}}_{=0(3)} = \underbrace{\cancel{\frac{\partial}{\partial t}}\int_{CV} e\,\rho\,dV\!\!\!\!/}_{=0(4)} + \int_{CS} (e + pv)\,\rho \vec{V}\cdot d\vec{A}$$

$$e = u + \frac{V^2}{2} + gz$$

Restrictions:
(1) $\dot{W}_s = 0$.
(2) $\dot{W}_{\text{shear}} = 0$.
(3) $\dot{W}_{\text{other}} = 0$.
(4) Steady flow.
(5) Uniform flow and properties at each section.



- **Bernoulli's Equation interpreted as an Energy Equation:**
- An equation identical in form to Bernoulli's equation (although requiring very different restrictions) may be obtained from the first law of thermodynamics.

$$\left(u_1 + p_1 v_1 + \frac{V_1^2}{2} + g z_1\right)(-\rho_1 V_1 A_1) + \left(u_2 + p_2 v_2 + \frac{V_2^2}{2} + g z_2\right)(\rho_2 V_2 A_2) - \dot{Q} = 0$$

But from continuity under these restrictions,

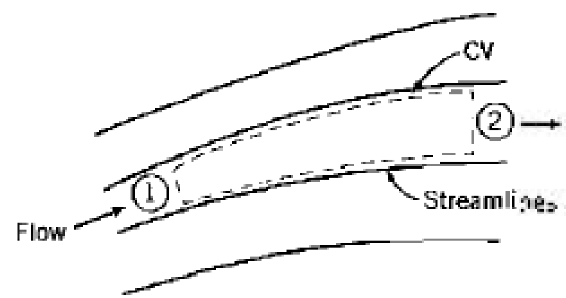
$$\underbrace{\frac{\partial}{\partial t}\int_{CV} \rho \, d\forall}_{=0(4)} + \int_{CS} \rho \, \vec{V} \cdot d\vec{A} = 0$$

$$(-\rho_1 V_1 A_1) + (\rho_2 V_2 A_2) = 0$$

$$\dot{m} = \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

$$\dot{Q} = \frac{\delta Q}{dt} = \frac{\delta Q}{dm}\frac{dm}{dt} = \frac{\delta Q}{dm}\dot{m}$$

Restrictions:
(1) $\dot{W}_s = 0$.
(2) $\dot{W}_{shear} = 0$.
(3) $\dot{W}_{other} = 0$.
(4) Steady flow.
(5) Uniform flow and properties at each section.



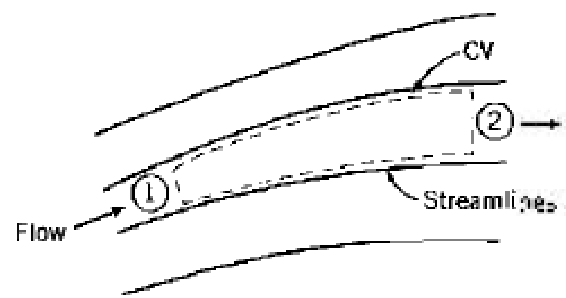
- **Bernoulli's Equation interpreted as an Energy Equation:**
- An equation identical in form to Bernoulli's equation (although requiring very different restrictions) may be obtained from the first law of thermodynamics.

Thus, from the energy equation,

$$\left[\left(p_2 v_2 + \frac{V_2^2}{2} + gz_2\right) - \left(p_1 v_1 + \frac{V_1^2}{2} + gz_1\right)\right]\dot{m} + \left(u_2 - u_1 - \frac{\delta Q}{dm}\right)\dot{m} = 0$$

$$p_1 v_1 + \frac{V_1^2}{2} + gz_1 = p_2 v_2 + \frac{V_2^2}{2} + gz_2 + \left(u_2 - u_1 - \frac{\delta Q}{dm}\right)$$

Restrictions:
(1) $\dot{W}_s = 0$.
(2) $\dot{W}_{shear} = 0$.
(3) $\dot{W}_{other} = 0$.
(4) Steady flow.
(5) Uniform flow and properties at each section.

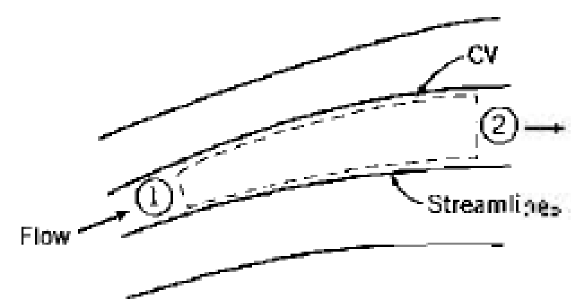


- **Bernoulli's Equation interpreted as an Energy Equation:**
- An equation identical in form to Bernoulli's equation (although requiring very different restrictions) may be obtained from the first law of thermodynamics.

Under the additional assumption (6) of incompressible flow, $v_1 = v_2 = 1/\rho$ and hence

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 + \left(u_2 - u_1 - \frac{\delta Q}{dm}\right)$$

Restrictions: (1) $\dot{W}_s = 0$.
(2) $\dot{W}_{shear} = 0$.
(3) $\dot{W}_{other} = 0$.
(4) Steady flow.
(5) Uniform flow and properties at each section.



- **Bernoulli's Equation interpreted as an Energy Equation:**
- The above Equation would reduce to the Bernoulli equation if the term in parentheses were zero.
- Thus, under the further restriction, (7) $(u_2 - u_1 - \delta Q/dm) = 0$

the energy equation reduces to

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

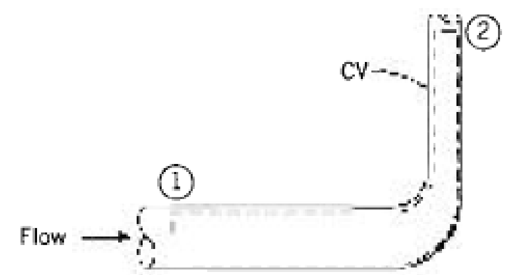
- The above Equation is identical in form to the Bernoulli equation.
- The Bernoulli equation was derived from momentum considerations (Newton's second law), and is valid for steady, incompressible, frictionless flow along a streamline.
- The above Equation was obtained by applying the first law of thermodynamics to a stream tube control volume, subject to restrictions 1 through 7 above.

- **Energy Considerations in Pipe Flow:**
- Consider, for example, steady flow through the piping system, including a reducing elbow, shown in Fig.
- The control volume boundaries are shown as dashed lines.
- They are normal to the flow at sections 1 and 2 and coincide with the inside surface of the pipe wall elsewhere.

Basic equation:

$$\dot{Q} - \cancelto{0(1)}{\dot{W}_s} - \cancelto{0(2)}{\dot{W}_{shear}} - \cancelto{0(1)}{\dot{W}_{other}} = \cancelto{0(3)}{\frac{\partial}{\partial t}\int_{CV} e\,\rho\, dV} + \int_{CS} (e + pv)\,\rho \vec{V} \cdot d\vec{A}$$

$$e = u + \frac{V^2}{2} + gz$$



Assumptions: (1) $\dot{W}_s = 0$, $\dot{W}_{other} = 0$.
(2) $\dot{W}_{shear} = 0$ (although shear stresses are present at the walls of the elbow, the velocities are zero there).
(3) Steady flow.
(4) Incompressible flow.
(5) Internal energy and pressure uniform across sections 1 and 2.

- **Energy Considerations in Pipe Flow:**
- Consider, for example, steady flow through the piping system, including a reducing elbow, shown in Fig.

Under these assumptions the energy equation reduces to

$$\dot{Q} = \dot{m}(u_2 - u_1) + \dot{m}\left(\frac{p_2}{\rho} - \frac{p_1}{\rho}\right) + \dot{m}g(z_2 - z_1)$$

$$+ \int_{A_2} \frac{V_2^2}{2} \rho V_2 \, dA_2 - \int_{A_1} \frac{V_1^2}{2} \rho V_1 \, dA_1$$

- **Energy Considerations in Pipe Flow:**
- Since we know that for viscous flows the velocity at a cross-section cannot be uniform.
- However, it is convenient to introduce the average velocity into Eq. so that we can eliminate the integrals.
- To do this, we define a kinetic energy coefficient.

The *kinetic energy coefficient*, α, is defined such that

$$\int_A \frac{V^2}{2} \rho V \, dA = \alpha \int_A \frac{\bar{V}^2}{2} \rho V \, dA = \alpha \dot{m} \frac{\bar{V}^2}{2}$$

$$\alpha = \frac{\int_A \rho V^3 \, dA}{\dot{m} \bar{V}^2}$$

- We can think of α as a correction factor that allows us to use the average velocity ύ to compute the kinetic energy at a cross section.
- For laminar flow in a pipe, α = 2.0.

- **Energy Considerations in Pipe Flow:**
- For Turbulent Flow, the equation is:

$$\alpha = \left(\frac{U}{\bar{V}}\right)^3 \frac{2n^2}{(3+n)(3+2n)}$$

The value of $\bar{V}/U$ is given by Eq. 8.24. For $n = 6$, $\alpha = 1.08$ and for $n = 10$, $\alpha = 1.03$. Since the exponent, n, in the power-law profile is a function of Reynolds number, α also varies with Reynolds number. Because α is reasonably close to unity for high Reynolds numbers, and because the change in kinetic energy is usually small compared with the dominant terms in the energy equation, *we shall almost always use the approximation $\alpha = 1$ in our pipe flow calculations*.

- **Kinetic Energy Correction Factor, *α*:**
- Kinetic energy correction factor is defined as the ratio of the kinetic energy of the flow per second based on actual velocity across a section to the kinetic energy oo the flow per second on average velocity across the same section.
- Hence mathematically,

$$\alpha = \frac{\text{K.E./sec based on actual velocity}}{\text{K.E./sec based on average velocity}}$$

- Prove $\alpha = 2$
- Kinetic energy of the fluid flowing through the elementary ring of radius r and of width dr per sec.

$$= \frac{1}{2} \times \text{mass} \times u^2 = \frac{1}{2} \times \rho dQ \times u^2$$

$$= \frac{1}{2} \times \rho \times (u \times 2\pi r\, dr) \times u^2 = \frac{1}{2}\rho \times 2\pi r u^3\, dr = \pi \rho r u^3\, dr$$

$$dA = 2\pi r\, dr$$

Rate of fluid flowing through the ring

$$= dQ = \text{velocity} \times \text{area of ring element}$$

$$= u \times 2\pi r\, dr$$

- **Kinetic Energy Correction Factor, *α*:**
- Kinetic energy correction factor is defined as the ratio of the kinetic energy of the flow per second based on actual velocity across a section to the kinetic energy oo the flow per second on average velocity across the same section.

$$u = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)(R^2 - r^2)$$

Total actual kinetic energy of flow per second

$$= \int_0^R \pi\rho r u^3\, dr = \int_0^R \pi\rho r \left[\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\left(R^2 - r^2\right)\right]^3 dr$$

$$= \pi\rho \times \left[\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\right]^3 \int_0^R [R^2 - r^2]^3\, r dr$$

$$= \pi\rho \times \frac{1}{64\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 \int_0^R (R^6 - r^6 - 3R^4r^2 + 3R^2r^4) r\, dr$$

$$= \frac{\pi\rho}{64\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 \int_0^R (R^6 r - r^7 - 3R^4r^3 + 3R^2r^5)\, dr$$

- **Kinetic Energy Correction Factor, *α*:**
- Kinetic energy correction factor is defined as the ratio of the kinetic energy of the flow per second based on actual velocity across a section to the kinetic energy oo the flow per second on average velocity across the same section.

$$= \frac{\pi\rho}{64\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3\left[\frac{R^6 r^2}{2}-\frac{r^8}{8}-\frac{3R^4 r^4}{4}+\frac{3R^2 r^6}{6}\right]_0^R$$

$$= \frac{\pi\rho}{64\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3\left[\frac{R^8}{2}-\frac{R^8}{8}-\frac{3R^8}{4}+\frac{3R^8}{6}\right]$$

$$= \frac{\pi\rho}{64\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 R^8\left[\frac{12-3-18+12}{24}\right]$$

$$= \frac{\pi\rho}{64\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 \frac{R^8}{8}$$

- **Kinetic Energy Correction Factor, α:**
- Kinetic energy correction factor is defined as the ratio of the kinetic energy of the flow per second based on actual velocity across a section to the kinetic energy oo the flow per second on average velocity across the same section.

Kinetic energy of the flow based on average velocity

$$= \frac{1}{2} \times \text{mass} \times \bar{u}^2 = \frac{1}{2} \times \rho A \bar{u} \times \bar{u}^2 = \frac{1}{2} \times \rho A \bar{u}^3$$

Substituting the value of $A = \pi R^2$

ıd $$\bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^2$$

- **Kinetic Energy Correction Factor, *α*:**
- Kinetic energy correction factor is defined as the ratio of the kinetic energy of the flow per second based on actual velocity across a section to the kinetic energy oo the flow per second on average velocity across the same section.

Kinetic energy of the flow/sec

$$= \frac{1}{2} \times \rho \times \pi R^2 \times \left[\frac{1}{8\mu} \left(-\frac{\partial p}{\partial x} \right) R^2 \right]^3$$

$$= \frac{1}{2} \times \rho \times \pi R^2 \times \frac{1}{64 \times 8\mu^3} \left(-\frac{\partial p}{\partial x} \right)^3 \times R^6$$

$$= \frac{\rho \pi}{128 \times 8\mu^3} \left(-\frac{\partial p}{\partial x} \right)^3 \times R^8$$

- **Kinetic Energy Correction Factor, *α*:**
- Kinetic energy correction factor is defined as the ratio of the kinetic energy of the flow per second based on actual velocity across a section to the kinetic energy oo the flow per second on average velocity across the same section.

$$\alpha = \frac{\text{K.E./sec based on actual velocity}}{\text{K.E./sec based on average velocity}}$$

$$= \frac{\dfrac{\pi\rho}{64\mu^3}\left(-\dfrac{\partial p}{\partial x}\right)^3 \times \dfrac{R^8}{8}}{\dfrac{\rho}{128 \times 8\mu^3}\left(-\dfrac{\partial p}{\partial x}\right)^3 \times R^8} = \frac{128 \times 8}{64 \times 8} = \mathbf{2.0.\ Ans.}$$

- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**

- **<u>Reference:</u>**
- Fluid Mechanics by Fox
- Fluid Mechanics by Bansal
- Fluid Mechanics by Young
- NPTEL