# CL24203 – FLUID MECHANICS

**III Semester BTech (Chemical Engineering)**

# CL24203 FLUID MECHANICS



- **Module 2:**
- **Fluid flow phenomena:**
- **Fluid as a continuum,**
- Terminologies of fluid flow, velocity – local, average, maximum,
- **flow rate – mass, volumetric,** velocity field;
- **dimensionality of flow;**
- flow visualization – streamline, path line, streak line, stress field;
- viscosity;
- Newtonian fluid; Non-Newtonian fluid;
- Reynolds number & its significance,
- **laminar, transition and turbulent flows:** Prandtl boundary layer,
- **compressible and incompressible.**
- Momentum equation for integral control volume,
- momentum correction factor. [8]

# LECTURE PLAN AND LEARNING OBJECTIVES FOR 40 [ONE HOUR] LECTURES

**Module 2:** Fluid flow phenomena: Fluid as a continuum, Terminologies of fluid flow, velocity – local, average, maximum, flow rate – mass, volumetric, velocity field; dimensionality of flow; flow visualization – streamline, path line, streak line, stress field; viscosity; Newtonian fluid; Non-Newtonian fluid; Reynolds number its significance, laminar, transition and turbulent flows: Prandtl boundary layer, compressible and incompressible. Momentum equation for integral control volume, momentum correction factor. [8]

**Lecture I**

Fluid flow phenomena: Fluid as a continuum.

**Lecture II**

Terminologies of fluid flow, velocity – local, average, maximum, flow rate – mass, volumetric, velocity field.

**Lecture III**

Terminologies of fluid flow, dimensionality of flow; flow visualization – streamline, path line, streak line, stress field.

**Lecture IV**

Terminologies of fluid flow, viscosity; Newtonian fluid; Non-Newtonian fluid.

**Lecture V**

Reynolds number and its significance.

# LECTURE PLAN AND LEARNING OBJECTIVES FOR 40 [ONE HOUR] LECTURES

**Module 2:** Fluid flow phenomena: Fluid as a continuum, Terminologies of fluid flow, velocity – local, average, maximum, flow rate – mass, volumetric, velocity field; dimensionality of flow; flow visualization – streamline, path line, streak line, stress field; viscosity; Newtonian fluid; Non-Newtonian fluid; Reynolds number its significance, laminar, transition and turbulent flows: Prandtl boundary layer, compressible and incompressible. Momentum equation for integral control volume, momentum correction factor. [8]

**Lecture VI**

Laminar, transition and turbulent flows: Prandtl boundary layer, compressible and incompressible.

**Lecture VII**

Laminar, transition and turbulent flows: Prandtl boundary layer, compressible and incompressible.

**Lecture VIII**

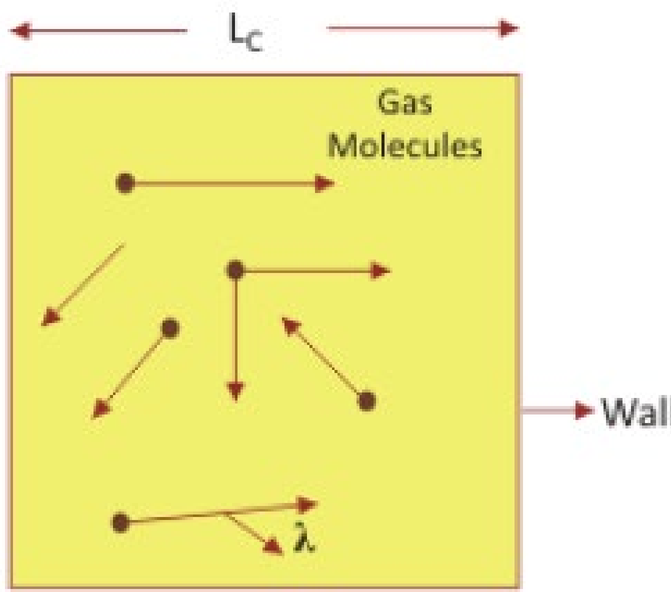
Momentum equation for integral control volume, momentum correction factor.

- **Fluid as a continuum or continuum based approach:**
- Fluid is made of molecules.
- However, for most of the engineering applications, when we speak of fluid's properties such as density, or conditions such as pressure and temperature, we do not imply such properties or conditions of individual molecules, but those of "fluid" as a whole.
- In other words, we refer to the average or macroscopic aggregate effects of the fluid-molecules, reflected in pressure, temperature, density, etc.
- Such an approach to treating a fluid is called continuum based approach. In other words, fluid is treated as continuum.
- However, there is a restriction.
- The continuum approach can be applied only when the mean free path of the fluid (largely, gas) is smaller (actually much smaller!!) than the physical characteristic length of the system under consideration, say, the diameter of the tube in which the gas flows, or size of a container in which gas is stored.

- **Fluid as a continuum or continuum based approach:**

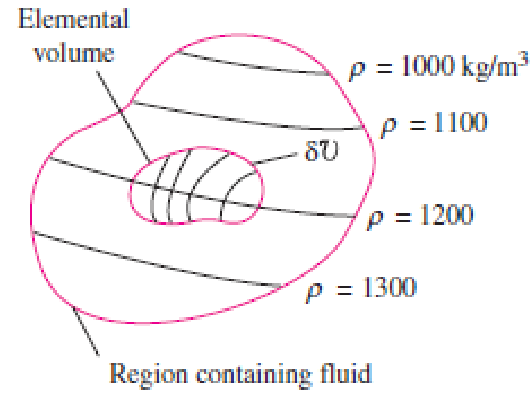
Mathematically, for the continuum approach based model to hold good, where $\lambda$ is the mean free path of the gas molecule and $L_C$ is the characteristic length of the system. Alternatively, Knudsen #defined as $\lambda / L_C << 1$.

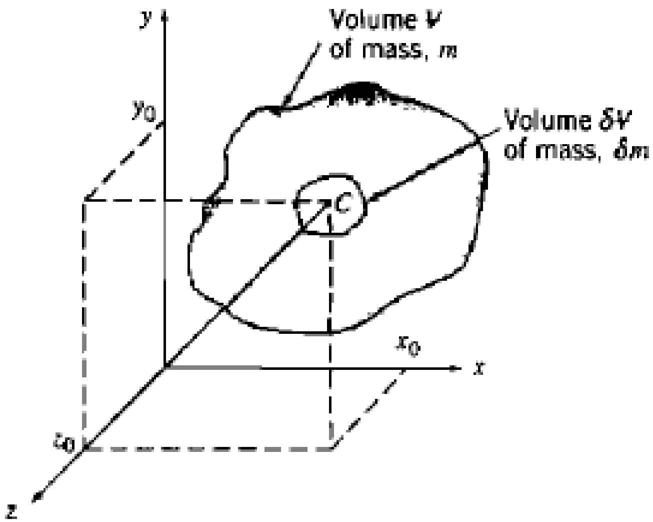


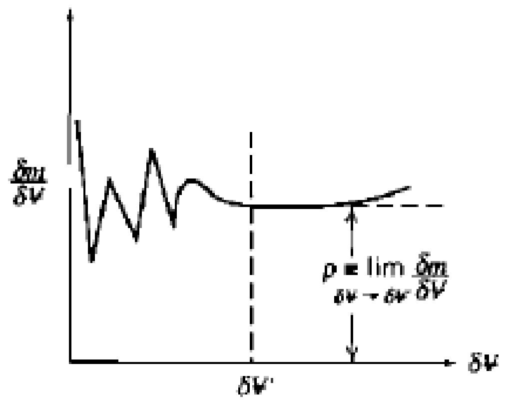

- In this section we will discuss under what circumstances a fluid can be treated as a continuum, for which, by definition, properties vary smoothly from point to point.

- **Fluid as a continuum or continuum based approach:**
- The concept of a continuum is the basis of classical fluid mechanics.
- The continuum assumption is valid in treating the behavior of fluids under normal conditions.
- As a consequence of the continuum assumption, each fluid property is assumed to have a definite value at every point in space.
- Thus fluid properties such as density, temperature, velocity, and so on, are considered to be continuous functions of position and time.
- To illustrate the concept of a property at a point, consider how we determine the density at a point.
- A region of fluid is shown in Fig.
- We are interested in determining the density at the point C, whose coordinates are x0, y0, and Z0.
- Density is defined as mass per unit volume.
- Thus the average density in volume V is given by $\rho = m/V$.
- In general, because the density of the fluid may not be uniform, this will not be equal to the value of the density at point C.

- **Fluid as a continuum or continuum based approach:**
- To determine the density at point C, we must select a small volume, δV, surrounding point C and then determine the ratio δm/δV.
- The question is, how small can we make the volume δV?
- We can answer this question by plotting the ratio δm/δV, and allowing the volume to shrink continuously in size.



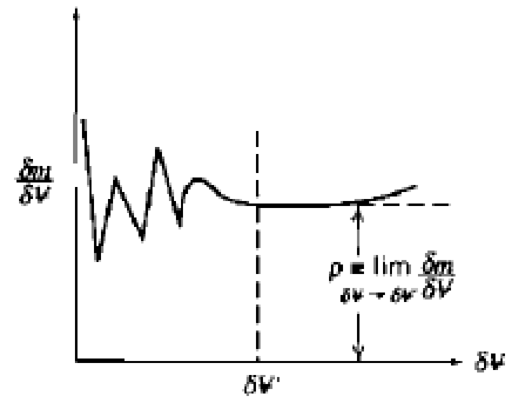



- **Fluid as a continuum or continuum based approach:**
- Assuming that volume δV is initially relatively large (but still small compared with the volume, V) a typical plot of δm/δV might appear as in Fig. b.
- In other words, δV must be sufficiently large to yield a meaningful, reproducible value for the density at a location and yet small enough to be called a point.
- The average density tends to approach an asymptotic value as the volume is shrunk to enclose only homogeneous fluid in the immediate neighborhood of point C.
- If δV becomes so small that it contains only a small number of molecules, it becomes impossible to fix a definite value for δm/δV; the value will vary erratically as molecules cross into and out of the volume.
- Thus there is a lower limiting value of δV, designated δV* in Fig. b, allowable for use in defining fluid density at a point.
- The density at a "point" is then defined as

$$\rho = \lim_{\delta V \to \delta V'} \frac{\delta m}{\delta V}$$



- **Fluid as a continuum or continuum based approach:**
- Since point C was arbitrary, the density at any other point in the fluid could be determined in the same manner.
- If density was measured simultaneously at an infinite number of points in the fluid, we would obtain an expression for the density distribution as a function of the space coordinates, $\rho = \rho(x, y, z)$, at the given instant.
- The density at a point may also vary with time (as a result of work done on or by the fluid and/or heat transfer to the fluid).
- Thus the complete representation of density (the field representation) is given by

$$\rho = \rho(x, y, z, t)$$

- Most engineering problems are concerned with physical dimensions much larger than this limiting volume, so that density is essentially a point function and fluid properties can be thought of as varying continually in space, as sketched in Fig.
- Such a fluid is called a continuum, which simply means that its variation in properties is so smooth that differential calculus can be used to analyze the substance.

- **Methods of describing Fluid Motion:**
- The analysis of fluid mechanics problem is approached by two methods: (a) Lagrangian Method and (b) Eulerian Method.
- In the Lagrangian method, an individual fluid particle is followed as they move about and determining how the fluid properties associated with these particles change as a function of time.
- In the Eulerian method, the fluid particles such as velocity, acceleration, pressure, density etc., are described at a point in flow field as functions of space and time.
- From this method we obtain information about the flow in terms of what happens at fixed points in space as the fluid flows past those points.
- For e.g., In the Eulerian method we compute the pressure field p(x, y, z, t) of the flow pattern, not the pressure changes p(t) that a particle experiences as it moves through the field.
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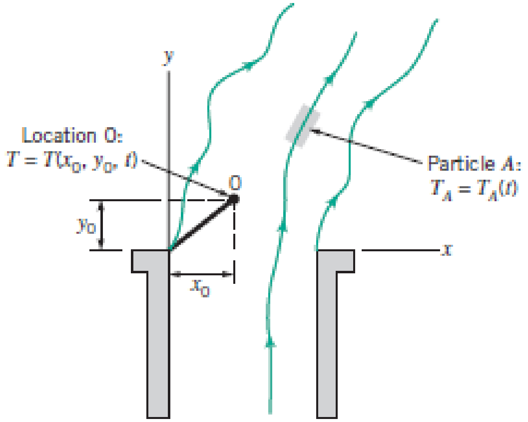
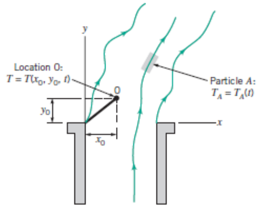


- **Methods of describing Fluid Motion:**
- The difference between the two methods of analyzing fluid problems can be seen in the example of smoke discharging from a chimney, as is shown in Fig.
- In the Eulerian method one may attach a temperature-measuring device to the top of the chimney (point 0) and record the temperature at that point as a function of time.
- That is, T = T(x0, y0, z0, t).
- The use of numerous temperature-measuring devices fixed at various locations would provide the temperature field, T = T(x, y, z, t).
- In the Lagrangian method, one would attach the temperature-measuring device to a particular fluid particle (particle A) and record that particle's temperature as it moves about.
- Thus, one would obtain that particle's temperature as a function of time, TA = TA(t).
- The use of many such measuring devices moving with various fluid particles would provide the temperature of these fluid particles as a function of time.
- In fluid mechanics it is usually easier to use the Eulerian method to describe a flow.



- **Flow rate – Mass, Volumetric or Discharge:**
- Flow rate is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel.
- For an incompressible fluid, the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second.
- For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section.
- The relationship between volume flow and mass flow follows directly from the definition of density, ρ.

$$\rho = \frac{mass}{volume} \qquad \frac{\text{kg}}{\text{m}^3}$$

$$\rho = \frac{mass\ flow}{volume\ flow} = \frac{M}{Q} \qquad \frac{\text{kg}}{\text{s}}\frac{\text{s}}{\text{m}^3}$$

$$M = Q\rho \quad \text{kg/s}$$

- This is the simplest form of Continuity Equation.

- **Flow rate – Mass, Volumetric or Discharge:**
- Flow rate is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel.
- In other ways, a common method of measuring volume flow is to determine the average velocity of fluid across the section.
- Consider a liquid flowing through a pipe in which
- A = Cross-sectional area of pipe
- V = Average velocity of fluid across the section.
- Then discharge, Q = V x A $\quad \frac{m}{s}\ m^2 \quad \text{or} \quad \frac{m^3}{s}$
- Then Continuity Equation becomes:
- **M = ρ x V x A**

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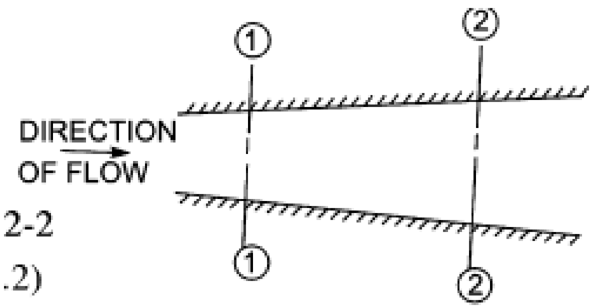
- **Continuity Equation:**
- The equation based on the principle of conservation of mass is called continuity equation.
- Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant.
- Consider two cross-sections of a pipe as shown in the Fig.
- Let $V_1$ = Average velocity at cross-section 1-1

  $\rho_1$ = Density at section 1-1

  $A_1$ = Area of pipe at section 1-1




2-2

.2)

and $V_2$, $\rho_2$, $A_2$ are corresponding values at section, 2-2.

Then rate of flow at section 1-1 = $\rho_1 A_1 V_1$

Rate of flow at section 2-2 = $\rho_2 A_2 V_2$

According to law of conservation of mass

Rate of flow at section 1-1 = Rate of flow at section 2-2

or $$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad \text{...(5.2)}$$

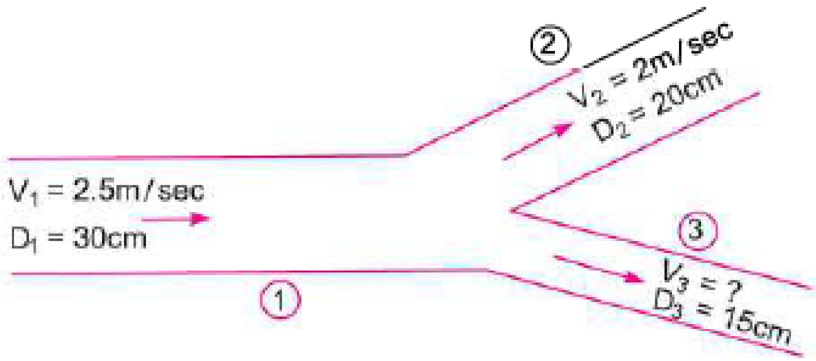
Equation (5.2) is applicable to the compressible as well as incompressible fluids and is called **Continuity Equation**. If the fluid is incompressible, then $\rho_1 = \rho_2$ and continuity equation (5.2) reduces to

$$A_1 V_1 = A_2 V_2$$

- **Continuity Equation:**
- The equation based on the principle of conservation of mass is called continuity equation.

**Problem 5.2** *A 30 cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.*

**Solution.** Given :




$D_1 = 30 \text{ cm} = 0.30 \text{ m}$

$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times .3^2 = 0.07068 \text{ m}^2$

$V_1 = 2.5 \text{ m/s}$

$D_2 = 20 \text{ cm} = 0.20 \text{ m}$

$A_2 = \frac{\pi}{4} (.2)^2 = \frac{\pi}{4} \times .4 = 0.0314 \text{ m}^2,$

$V_2 = 2 \text{ m/s}$

$D_3 = 15 \text{ cm} = 0.15 \text{ m}$

$A_3 = \frac{\pi}{4} (.15)^2 = \frac{\pi}{4} \times 0.225 = 0.01767 \text{ m}^2$

- **Continuity Equation:**
- The equation based on the principle of conservation of mass is called continuity equation.

**Problem 5.2** *A 30 cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.*

Find (*i*) Discharge in pipe 1 or $Q_1$

(*ii*) Velocity in pipe of dia. 15 cm or $V_3$

Let $Q_1$, $Q_2$ and $Q_3$ are discharges in pipe 1, 2 and 3 respectively.

Then according to continuity equation

$$Q_1 = Q_2 + Q_3$$

(*i*) **The discharge $\mathbf{Q_1}$** in pipe 1 is given by

$$Q_1 = A_1V_1 = 0.07068 \times 2.5 \text{ m}^3\text{/s} = \mathbf{0.1767 \text{ m}^3\text{/s. Ans.}}$$

(*ii*) **Value of $\mathbf{V_3}$**

$$Q_2 = A_2V_2 = 0.0314 \times 2.0 = 0.0628 \text{ m}^3\text{/s}$$

Substituting the values of $Q_1$ and $Q_2$ in equation (1)

$$0.1767 = 0.0628 + Q_3$$

$\therefore$ $$Q_3 = 0.1767 - 0.0628 = 0.1139 \text{ m}^3\text{/s}$$

But $$Q_3 = A_3 \times V_3 = 0.01767 \times V_3 \quad \text{or} \quad 0.1139 = 0.01767 \times V_3$$

$\therefore$ $$V_3 = \frac{0.1139}{0.01767} = \mathbf{6.44 \text{ m/s. Ans.}}$$

- ## Continuity Equation in Three Dimension:

Consider a fluid element of lengths $dx$, $dy$ and $dz$ in the direction of $x$, $y$ and $z$. Let $u$, $v$ and $w$ are the inlet velocity components in $x$, $y$ and $z$ directions respectively. Mass of fluid entering the face $ABCD$ per second

$$= \rho \times \text{Velocity in } x\text{-direction} \times \text{Area of } ABCD$$

$$= \rho \times u \times (dy \times dz)$$

Then mass of fluid leaving the face $EFGH$ per second $= \rho u\, dydz + \frac{\partial}{\partial x}(\rho u\, dydz)\, dx$

$\therefore$ Gain of mass in $x$-direction

$$= \text{Mass through } ABCD - \text{Mass through } EFGH \text{ per second}$$

$$= \rho u\, dydz - \rho u\, dydz - \frac{\partial}{\partial x}(\rho u\, dydz) dx$$

$$= -\frac{\partial}{\partial x}(\rho u\, dydz)\, dx$$

{$\because$ $dydz$ is constant}

$$= -\frac{\partial}{\partial x}(\rho u)\, dx\, dydz$$

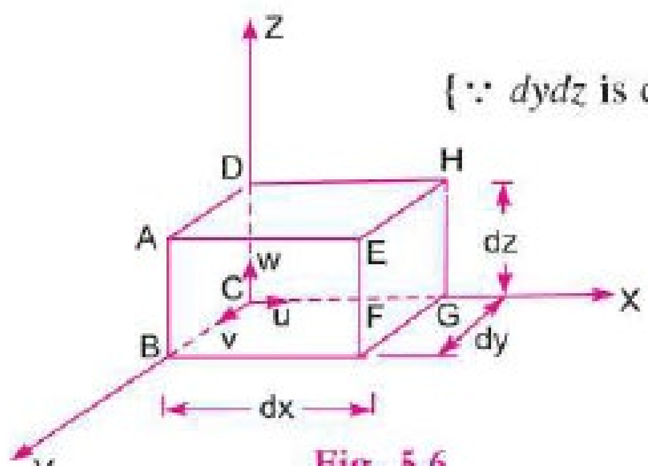



Fig. 5.6

- **Continuity Equation in Three Dimension:**

Similarly, the net gain of mass in *y*-direction

$$= -\frac{\partial}{\partial y}(\rho v)\, dxdydz$$

and in *z*-direction

$$= -\frac{\partial}{\partial z}(\rho w)\, dxdydz$$

$$\text{Net gain of masses} = -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dxdydz$$

Since the mass is neither created nor destroyed in the fluid element, the net increase of mass per unit time in the fluid element must be equal to the rate of increase of mass of fluid in the element. But mass of fluid in the element is $\rho.\ dx.\ dy.\ dz$ and its rate of increase with time is $\frac{\partial}{\partial t}(\rho\ dx.\ dy.\ dz)$ or

$$\frac{\partial \rho}{\partial t}.\ dx\ dy\ dz.$$

- **Continuity Equation in Three Dimension:**

Equating the two expressions,

or $$-\left[\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)\right] dxdydz = \frac{\partial \rho}{\partial t}.\ dxdydz$$

or $$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w) = 0$$ [Cancelling $dx.dy.dz$ from both sides] ...(5.3*A*)

Equation (5.3*A*) is the continuity equation in cartesian co-ordinates in its most general form. This equation is applicable to :

(*i*) Steady and unsteady flow,
(*ii*) Uniform and non-uniform flow, and
(*iii*) Compressible and incompressible fluids.

For steady flow, $\frac{\partial \rho}{\partial t} = 0$ and hence equation (5.3*A*) becomes as

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w) = 0 \qquad ...(5.3B)$$

If the fluid is incompressible, then $\rho$ is constant and the above equation becomes as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = 0 \qquad ...(5.4)$$

Equation (5.4) is the continuity equation in three-dimensions. For a two-dimensional flow, the component $w = 0$ and hence continuity equation becomes as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0. \qquad ...(5.5)$$

- **Types of Fluid Flow:**
- Fluid flow is classified as:
- (i) Steady and Unsteady flows;
- (ii) Uniform and non-uniform flows;
- (iii) Laminar and turbulent flows;
- (iv) Compressible and incompressible flows;
- (v) Rotational and irrotational flows;
- (vi) One, two and three-dimensional flows.

- **Types of Fluid Flow:**
- **(i) Steady and Unsteady flows**
- It is defined as the type of flow in which the fluid characteristics like velocity, pressure, density, etc., at a point do not change with time.
- Thus for steady flow, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

- For Unsteady flow, the fluid characteristics at a point changes with respect to time.
- Thus for Unsteady flow, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} \neq 0, \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \text{ etc.}$$

- **Types of Fluid Flow:**
- **(ii) Uniform and non-uniform flows**
- It is defined as the type of flow in which the velocity at any given time does not change with respect to space (ie., length of direction of the flow).
- Thus for uniform flow, we have

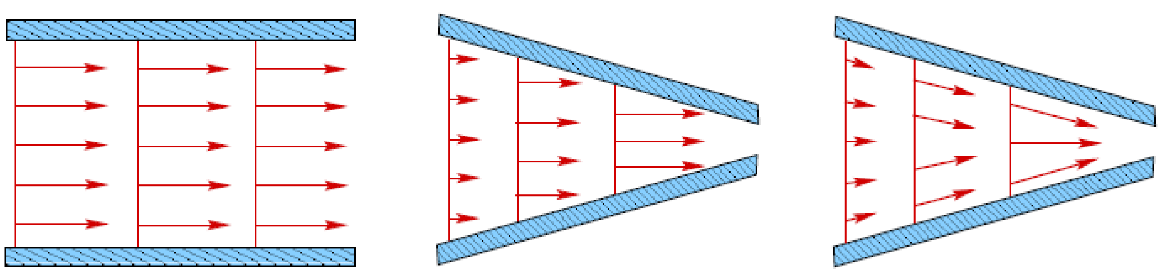
$$\left(\frac{\partial V}{\partial s}\right)_{t\,=\,\text{constant}} = 0$$

where $\partial V$ = Change of velocity

$\partial s$ = Length of flow in the direction $S$.

Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Thus, mathematically, for non-uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t\,=\,\text{constant}} \neq 0.$$



(a) (b) (c)

Figure 2.15: Uniform and non-uniform flows; (a) uniform flow, (b) non-uniform, but "locally uniform" flow, (c) non-uniform flow.

- **Types of Fluid Flow:**
- **(iii) Laminar and turbulent flows**
- Laminar flow is defined as the type of flow in which the fluid particles move along well-defined paths or stream line and all the stream-lines are straight and parallel.
- Thus the particles move in layers or laminas gliding smoothly over the adjacent layer.
- This type of flow is also called stream-line flow or viscous flow.

- Turbulent flow is the type of flow in which the fluid particles move in a zig-zag way.
- Due to the movement of fluid particles in a zig-zag way, the eddies formation takes place which are responsible for high energy loss.
- Basically, the type of low is determined by a non-dimensional number called the Reynold number.

If the Reynold number is less than 2000, the flow is called laminar. If the Reynold number is more than 4000, it is called turbulent flow. If the Reynold number lies between 2000 and 4000, the flow may be laminar or turbulent.

- Between 2000 and 4000, it is a transition region where the flow may be laminar or turbulent.

- **Types of Fluid Flow:**
- **(iii) Laminar and turbulent flows**
- Most common experience with the distinction between laminar and turbulent flow comes from observing the flow of water from a faucet as we increase the flow rate and shown in Fig.
- Part (a) of the figure displays a laminar (and steady) relatively low-speed flow in which the trajectories followed by fluid particles are very regular and smooth; furthermore, there is no indication that these trajectories might exhibit drastic changes in direction.

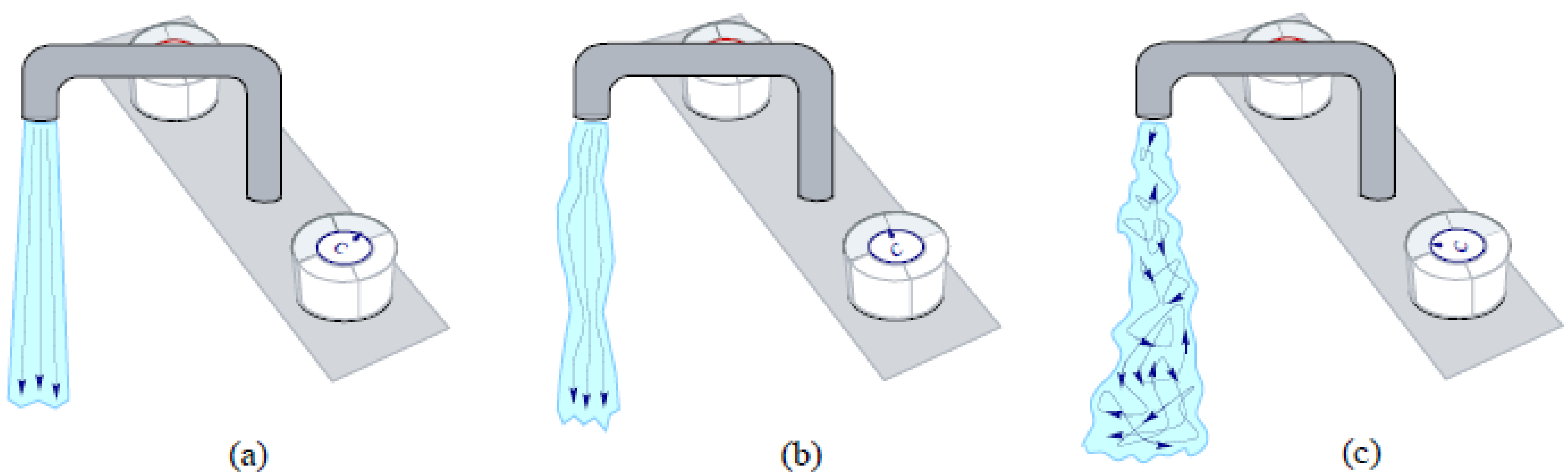



Figure 2.19: Laminar and turbulent flow of water from a faucet; (a) steady laminar, (b) periodic, wavy laminar, (c) turbulent.

- **Types of Fluid Flow:**
- **(iii) Laminar and turbulent flows**
- In part (b) of the figure we present a flow that is still laminar, but one that results as we open the faucet more than in the previous case, permitting a higher flow speed.
- In such a case the surface of the stream of water begins to exhibit waves, and these will change in time (basically in a periodic way).
- Thus the flow has become time dependent, but there is still no apparent intermingling of trajectories.
- Finally, in part (c) of the figure we show a turbulent flow corresponding to much higher flow speed.
- We see that the paths followed by fluid particles are now quite complicated and entangled indicating a high degree of mixing. Such flows are three dimensional and time dependent, and very difficult to predict in detail.

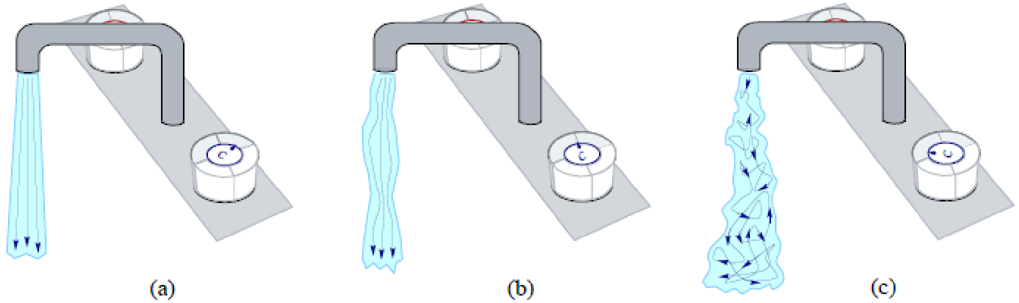
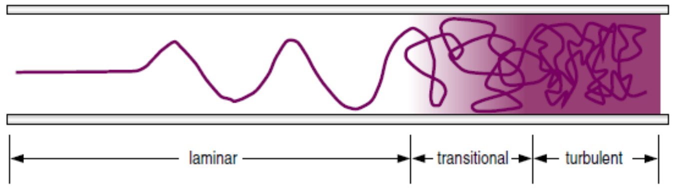



Figure 2.19: Laminar and turbulent flow of water from a faucet; (a) steady laminar, (b) periodic, wavy laminar, (c) turbulent.



- **Types of Fluid Flow:**
- **(iv) Compressible and incompressible flows**
- Compressible flow is the type of flow in which the density of the fluid changes from point to point or in other words the density is not constant for the fluid.
- Thus for compressible flow, we have

$$\rho \neq \text{Constant}$$

- Incompressible flow is the type of flow in which the density is constant for the fluid flow.
- Liquids are generally incompressible while gases are compressible.
- Thus for incompressible flow, we have.

$$\rho = \text{Constant.}$$

- **Types of Fluid Flow:**
- **(v) Rotational and irrotational flows**
- Rotational flow is the type of flow in which the fluid particles while flowing along steam-lines, also rotate about their own axis.

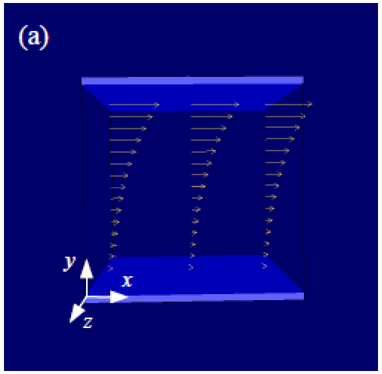
- And if the fluid particles while flowing along stream-lines, do not rotate about their own axis then the type of flow is called irrotational flow.

- **Types of Fluid Flow:**
- **(vi) One, two and three-dimensional flows: Dimensionality of flow**
- The dimensionality of a flow field corresponds to the number of spatial coordinates needed to describe all properties of the flow.
- One-dimensional flow is the type of flow in which the flow parameter such as velocity is a function of time and space co-ordinate only say x.
- For a steady one-dimensional flow, the velocity is a function of one-space-co-ordinate only.
- The variations of velocities in other two mutually perpendicular directions is assumed negligible.
- Thus for one-dimensional flow, we have

$$u = f(x),\ v = 0 \text{ and } w = 0$$

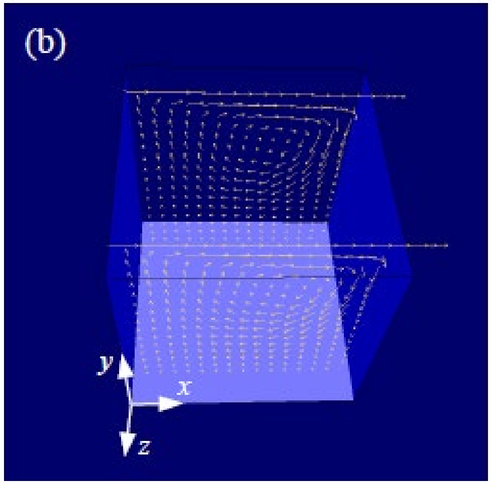
where $u$, $v$ and $w$ are velocity components in $x$, $y$ and $z$ directions respectively.

- Fluid flow is **three-dimensional** in nature.
- This means that the flow parameters like velocity, pressure and so on vary in all the
- three coordinate directions.
- Sometimes simplification is made in the analysis of different fluid flow problems



- **Types of Fluid Flow:**
- **(vi) One, two and three-dimensional flows: Dimensionality of flow**
- Two-dimensional flow is the type of flow in which the velocity is a function of time and two rectangular space co-ordinates say x and y.
- For a steady two-dimensional flow the velocity is a function of two space co-ordinates only.
- The variation of velocity in the third direction is negligible.
- Thus for one-dimensional flow, we have

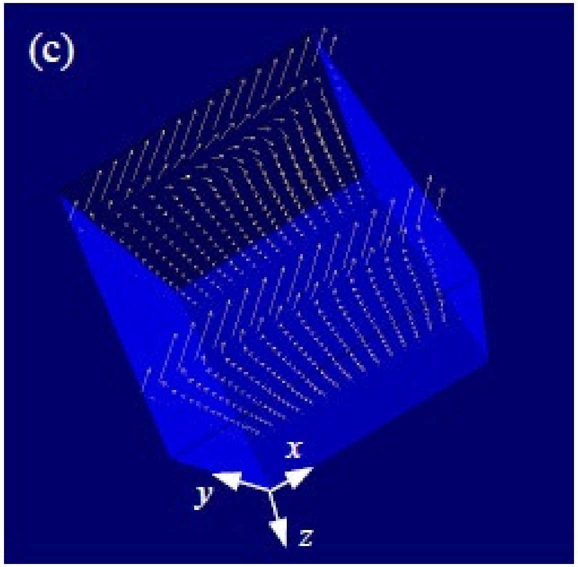
$$u = f_1(x, y),\ v = f_2(x, y) \text{ and } w = 0.$$



- **Types of Fluid Flow:**
- **(vi) One, two and three-dimensional flows: Dimensionality of flow**
- Three-dimensional flow is the type of flow in which the velocity is a function of time and three mutually perpendicular direcions.
- For a steady three-dimensional flow the velocity is a function of three space co-ordinates such as x, y and z only.
- Thus for one-dimensional flow, we have

$$u = f_1(x, y, z),\ v = f_2(x, y, z) \text{ and } w = f_3(x, y, z).$$



- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**

- **Reference:**
- Fluid Mechanics by Fox
- Fluid Mechanics by Bansal
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# CL203 – FLUID MECHANICS

**III Semester BTech (Chemical Engineering)**

# CL203 FLUID MECHANICS



- **Module 2:**
- **Fluid flow phenomena:**
- **Fluid as a continuum,**
- Terminologies of fluid flow, velocity – local, average, maximum,
- **flow rate – mass, volumetric, velocity field;**
- **dimensionality of flow;**
- flow visualization – streamline, path line, streak line, stress field;
- viscosity;
- Newtonian fluid; Non-Newtonian fluid;
- Reynolds number & its significance,
- **laminar, transition and turbulent flows:** Prandtl boundary layer,
- **compressible and incompressible.**
- Momentum equation for integral control volume,
- momentum correction factor. [8]

# LECTURE PLAN AND LEARNING OBJECTIVES FOR 40 [ONE HOUR] LECTURES

**Module 2:** Fluid flow phenomena: Fluid as a continuum, Terminologies of fluid flow, velocity – local, average, maximum, flow rate – mass, volumetric, velocity field; dimensionality of flow; flow visualization – streamline, path line, streak line, stress field; viscosity; Newtonian fluid; Non-Newtonian fluid; Reynolds number its significance, laminar, transition and turbulent flows: Prandtl boundary layer, compressible and incompressible. Momentum equation for integral control volume, momentum correction factor. [8]

**Lecture I**

Fluid flow phenomena: Fluid as a continuum.

**Lecture II**

Terminologies of fluid flow, velocity – local, average, maximum, flow rate – mass, volumetric, velocity field.

**Lecture III**

Terminologies of fluid flow, dimensionality of flow; flow visualization – streamline, path line, streak line, stress field.

**Lecture IV**

Terminologies of fluid flow, viscosity; Newtonian fluid; Non-Newtonian fluid.

**Lecture V**

Reynolds number and its significance.

# LECTURE PLAN AND LEARNING OBJECTIVES FOR 40 [ONE HOUR] LECTURES

**Module 2:** Fluid flow phenomena: Fluid as a continuum, Terminologies of fluid flow, velocity – local, average, maximum, flow rate – mass, volumetric, velocity field; dimensionality of flow; flow visualization – streamline, path line, streak line, stress field; viscosity; Newtonian fluid; Non-Newtonian fluid; Reynolds number its significance, laminar, transition and turbulent flows: Prandtl boundary layer, compressible and incompressible. Momentum equation for integral control volume, momentum correction factor. [8]

**Lecture VI**

Laminar, transition and turbulent flows: Prandtl boundary layer, compressible and incompressible.

**Lecture VII**

Laminar, transition and turbulent flows: Prandtl boundary layer, compressible and incompressible.

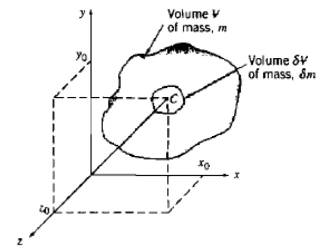
**Lecture VIII**

Momentum equation for integral control volume, momentum correction factor.

- **Velocity Field:**
- The representation of fluid parameters as functions of the spatial coordinates is termed a **field representation of the flow**. One of the most important fluid variables is the **velocity field.**
- In dealing with fluids in motion, we shall be concerned with the description of a velocity field.
- Refer to Fig. Define the fluid velocity at point C as the instantaneous velocity of the center of the volume, δV*, instantaneously surrounding point C.
- If we define a fluid particle as a small mass of fluid of fixed identity of volume δV*, then the velocity at point C is defined as the instantaneous velocity of the fluid particle which, at a given instant, is passing though point C.
- The velocity at any point in the flow field is defined similarly.
- At a given instant the velocity field, V, is a function of the space coordinates x, y, z.
- The velocity at any point in the flow field might vary from one instant to another.
- Thus the complete representation of velocity (the velocity field) is given by

$$\vec{V} = V(x, y, z, t)$$



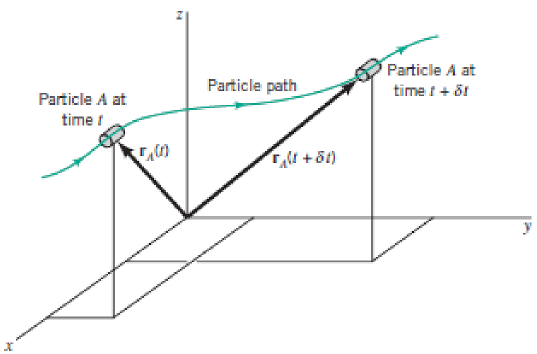
- **Velocity Field:**

Let $V$ is the resultant velocity at any point in a fluid flow. Let $u$, $v$ and $w$ are its component in $x$, $y$ and $z$ directions. The velocity components are functions of space-co-ordinates and time. Mathematically, the velocity components are given as

$$u = f_1(x, y, z, t)$$
$$v = f_2(x, y, z, t)$$
$$w = f_3(x, y, z, t)$$

and Resultant velocity, $V = ui + vj + wk = \sqrt{u^2 + v^2 + w^2}$

$$\mathbf{V} = u(x, y, z, t)\hat{\mathbf{i}} + v(x, y, z, t)\hat{\mathbf{j}} + w(x, y, z, t)\hat{\mathbf{k}}$$



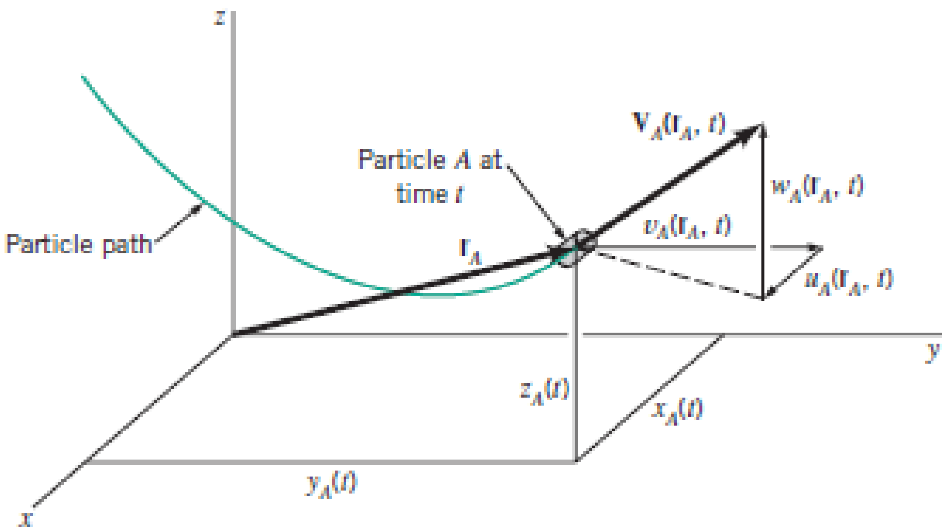

- By definition, the velocity of a particle is the time rate of change of the position vector for that particle.
- The position of particle A relative to the coordinate system is given by its position vector, $r_A$, which (if the particle is moving) is a function of time.
- The time derivative of this position gives the velocity of the particle, $dr_A/dt = V_A$.

- **Acceleration Field:**
- Describing Acceleration field as a function of position and time without actually following any particular particle.
- describing the flow in terms of the velocity field, V = V(x, y, z, t), rather than the velocity for particular particles.
- MATERIAL DERIVATIVE:

Consider a fluid particle moving along its pathline as is shown in Fig. 4.4. In general, the particle's velocity, denoted $\mathbf{V}_A$ for particle *A*, is a function of its location and the time.

$$\mathbf{V}_A = \mathbf{V}_A(\mathbf{r}_A, t) = \mathbf{V}_A[x_A(t), y_A(t), z_A(t), t]$$



- **Acceleration Field:**
- MATERIAL DERIVATIVE:

where $x_A = x_A(t)$, $y_A = y_A(t)$, and $z_A = z_A(t)$ define the location of the moving particle. By definition, the acceleration of a particle is the time rate of change of its velocity. Thus, we use the chain rule of differentiation to obtain the acceleration of particle $A$, $\mathbf{a}_A$, as

$$\mathbf{a}_A(t) = \frac{d\mathbf{V}_A}{dt} = \frac{\partial \mathbf{V}_A}{\partial t} + \frac{\partial \mathbf{V}_A}{\partial x}\frac{dx_A}{dt} + \frac{\partial \mathbf{V}_A}{\partial y}\frac{dy_A}{dt} + \frac{\partial \mathbf{V}_A}{\partial z}\frac{dz_A}{dt} \qquad (4.2)$$

Using the fact that the particle velocity components are given by $u_A = dx_A/dt$, $v_A = dy_A/dt$, and $w_A = dz_A/dt$, Eq. 4.2 becomes

$$\mathbf{a}_A = \frac{\partial \mathbf{V}_A}{\partial t} + u_A\frac{\partial \mathbf{V}_A}{\partial x} + v_A\frac{\partial \mathbf{V}_A}{\partial y} + w_A\frac{\partial \mathbf{V}_A}{\partial z}$$

Because the equation just described is valid for any particle, we can drop the reference to particle $A$ and obtain the acceleration field from the velocity field as

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z} \qquad (4.3)$$

- **Acceleration Field:**
- MATERIAL DERIVATIVE:

This is a vector result whose scalar components can be written as

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \quad (4.4)$$

and

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

The result given in Eq. 4.4 is often written in shorthand notation as

$$\mathbf{a} = \frac{D\mathbf{V}}{Dt}$$

where the operator

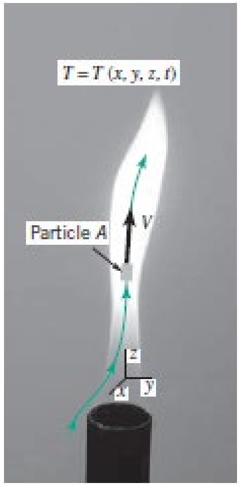
$$\frac{D(\ )}{Dt} \equiv \frac{\partial(\ )}{\partial t} + u\frac{\partial(\ )}{\partial x} + v\frac{\partial(\ )}{\partial y} + w\frac{\partial(\ )}{\partial z} \quad (4.5)$$

- is termed the ***material derivative*** or *substantial derivative.*

- **Acceleration Field:**
- MATERIAL DERIVATIVE:
- The material derivative concept is very useful in analysis involving various fluid parameters, not just the acceleration.
- The material derivative of any variable is the rate at which that variable changes with time for a given particle.
- For example, consider a temperature field

$T = T(x, y, z, t)$ associated with a given flow, like the flame shown in the figure in the margin. It may be of interest to determine the time rate of change of temperature of a fluid particle (particle $A$) as it moves through this temperature field. If the velocity, $\mathbf{V} = \mathbf{V}(x, y, z, t)$, is known, we can apply the chain rule to determine the rate of change of temperature as

$$\frac{dT_A}{dt} = \frac{\partial T_A}{\partial t} + \frac{\partial T_A}{\partial x}\frac{dx_A}{dt} + \frac{\partial T_A}{\partial y}\frac{dy_A}{dt} + \frac{\partial T_A}{\partial z}\frac{dz_A}{dt}$$




This can be written as

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}$$

- **Acceleration Field:**
- MATERIAL DERIVATIVE:

$$\mathbf{a} = \frac{d\mathbf{V}}{dt} = \underbrace{\frac{\partial \mathbf{V}}{\partial t}}_{\text{Local}} + \underbrace{\left( u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z} \right)}_{\text{Convective}}$$

The term $\partial \mathbf{V}/\partial t$ is called the *local acceleration,*

- Local acceleration is defined as the rate of increase of velocity with respect to time at a given point in a flow field.
- Convective acceleration is defined as the rate of change of velocity due to the change of position of fluid particles in a fluid flow.

- ## Acceleration Field:

**Problem 5.6** *The velocity vector in a fluid flow is given*

$$V = 4x^3 i - 10x^2 yj + 2tk.$$

*Find the velocity and acceleration of a fluid particle at (2, 1, 3) at time t = 1.*

**Solution.** The velocity components $u$, $v$ and $w$ are $u = 4x^3$, $v = -10x^2 y$, $w = 2t$

For the point (2, 1, 3), we have $x = 2$, $y = 1$ and $z = 3$ at time $t = 1$.

Hence velocity components at (2, 1, 3) are

$$u = 4 \times (2)^3 = 32 \text{ units}$$

$$v = -10(2)^2(1) = -40 \text{ units}$$

$$w = 2 \times 1 = 2 \text{ units}$$

$\therefore$ Velocity vector $V$ at (2, 1, 3) $= 32i - 40j + 2k$

or Resultant velocity $= \sqrt{u^2 + v^2 + w^2}$

$$= \sqrt{32^2 + (-40)^2 + 2^2} = \sqrt{1024 + 1600 + 4} = \mathbf{51.26 \text{ units. Ans.}}$$

**Acceleration** is given by equation (5.6)

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

- **Acceleration Field:**

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

Now from velocity components, we have

$$\frac{\partial u}{\partial x} = 12x^2,\ \frac{\partial u}{\partial y} = 0,\ \frac{\partial u}{\partial z} = 0 \text{ and } \frac{\partial u}{\partial t} = 0$$

$$\frac{\partial v}{\partial x} = -20xy,\ \frac{\partial v}{\partial y} = -10x^2,\ \frac{\partial v}{\partial z} = 0\ \frac{\partial v}{\partial t} = 0$$

$$\frac{\partial w}{\partial x} = 0,\ \frac{\partial w}{\partial y} = 0,\ \frac{\partial w}{\partial z} = 0 \text{ and } \frac{\partial w}{\partial t} = 2.1$$

Substituting the values, the acceleration components at (2, 1, 3) at time $t = 1$ are

$$a_x = 4x^3(12x^2) + (-10x^2y)(0) + 2t \times (0) + 0$$

$$= 48x^5 = 48 \times (2)^5 = 48 \times 32 = 1536 \text{ units}$$

$$a_y = 4x^3(-20xy) + (-10x^2y)(-10x^2) + 2t(0) + 0$$

$$= -80x^4y + 100x^4y$$

$$= -80(2)^4(1) + 100(2)^4 \times 1 = -1280 + 1600 = 320 \text{ units.}$$

$$a_z = 4x^3(0) + (-10x^2y)(0) + (2t)(0) + 2.1 = 2.0 \text{ units}$$

∴ Acceleration is $A = a_x i + a_y j + a_z k =$ **1536i + 320j + 2k. Ans.**

or Resultant $A = \sqrt{(1536)^2 + (320)^2 + (2)^2}$ units

$$= \sqrt{2359296 + 102400 + 4} = \mathbf{1568.9 \text{ units. Ans.}}$$

- **Stress Field:**
- In fluid mechanics, we will need to understand the forces act on fluid particles.
- Each fluid particle can experience: surface forces (pressure, friction) that are generated by contact with other particles or a solid surface; and body forces (such as gravity and electromagnetic) that are experienced throughout the particle.
- The gravitational body force acting on an element of volume, dV, is given by ρgdV, where ρ is the density (mass per unit volume) and g is the local gravitational acceleration.
- Surface forces on a fluid particle lead to stresses.
- The concept of stress is useful for describing how forces acting on the boundaries of a medium (fluid or solid) are transmitted throughout the medium.
- For example, when you stand on a diving board, stresses are generated within the board (deflected).
- On the other hand, when a body moves through a fluid, stresses are developed within the fluid.
- The difference between a fluid and a solid is, as we've seen, that stresses in a fluid are mostly generated by motion rather than by deflection.

- **Stress Field:**
- Imagine the surface of a fluid particle in contact with other fluid particles, and consider the contact force being generated between the particles.
- Consider a portion, δA, of the surface at some point C shown in Fig. 2.4.

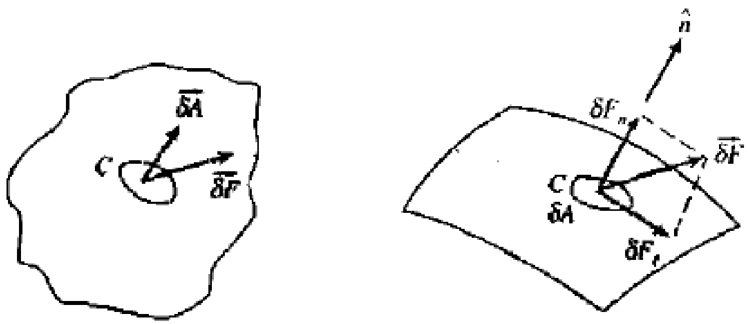
The force, $\delta F$, acting on $\delta A$ may be resolved into two components, one normal to and the other tangent to the area. A *normal stress* $\sigma_n$ and a *shear stress* $\tau_n$ are then defined as

$$\sigma_n = \lim_{\delta A_n \to 0} \frac{\delta F_n}{\delta A_n} \qquad (2.6)$$

and

$$\tau_n = \lim_{\delta A_n \to 0} \frac{\delta F_t}{\delta A_n} \qquad (2.7)$$

- The fluid is actually a continuum, so we could have imagined breaking it up any number of different ways into fluid particles around point C, and therefore obtained any number of different stresses at point C.



- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**

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# CL203 – FLUID MECHANICS

## III Semester BTech (Chemical Engineering)

# CL203 FLUID MECHANICS



- **Module 2:**
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- **viscosity;**
- **Newtonian fluid; Non-Newtonian fluid;**
- Reynolds number & its significance,
- **laminar, transition and turbulent flows:** Prandtl boundary layer,
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# LECTURE PLAN AND LEARNING OBJECTIVES FOR 40 [ONE HOUR] LECTURES

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**Lecture I**

Fluid flow phenomena: Fluid as a continuum.

**Lecture II**

Terminologies of fluid flow, velocity – local, average, maximum, flow rate – mass, volumetric, velocity field.

**Lecture III**

Terminologies of fluid flow, dimensionality of flow; flow visualization – streamline, path line, streak line, stress field.

**Lecture IV**

Terminologies of fluid flow, viscosity; Newtonian fluid; Non-Newtonian fluid.

**Lecture V**

Reynolds number and its significance.

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**Lecture VI**

Laminar, transition and turbulent flows: Prandtl boundary layer, compressible and incompressible.

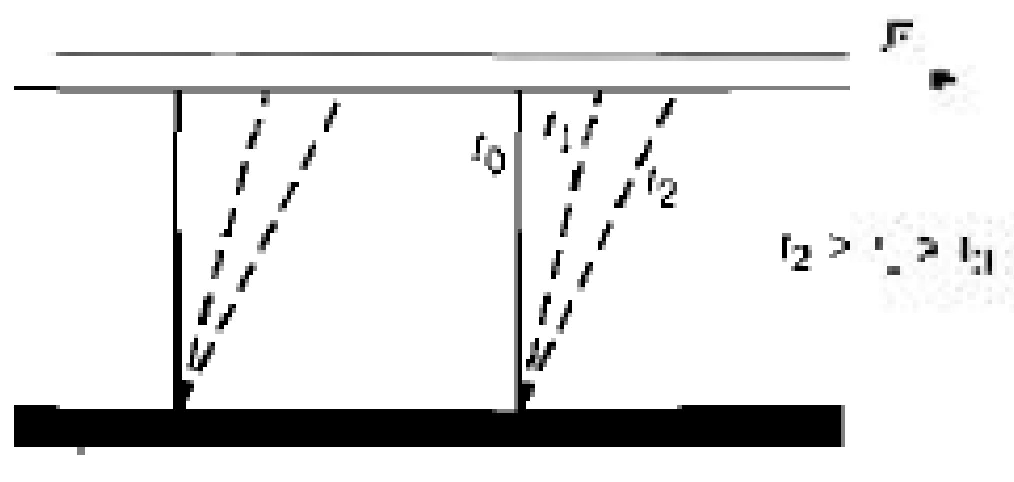
**Lecture VII**

Laminar, transition and turbulent flows: Prandtl boundary layer, compressible and incompressible.

**Lecture VIII**

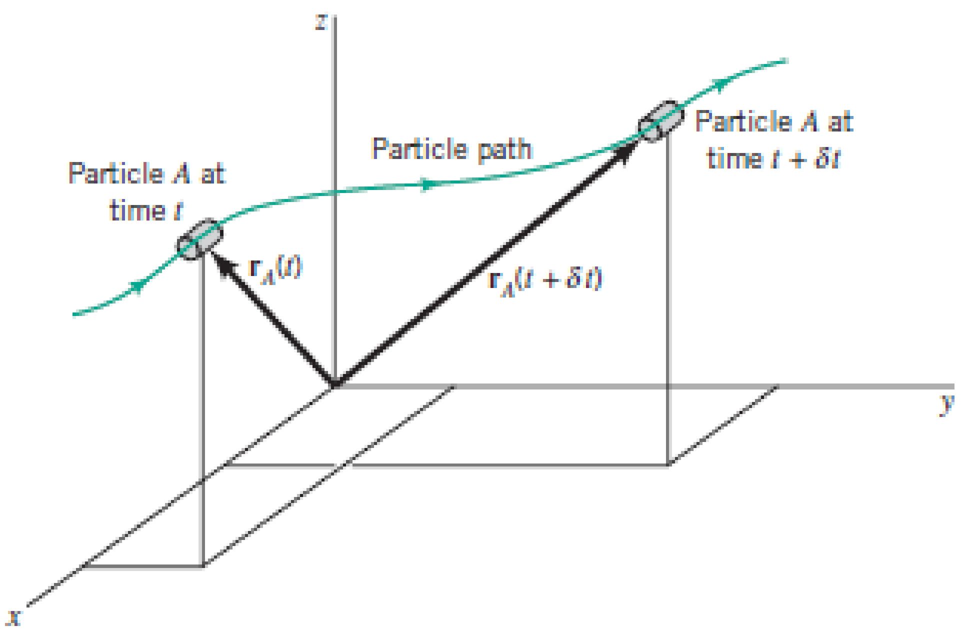
Momentum equation for integral control volume, momentum correction factor.

- **Flow Visualizaion – Stream line, Path line and Streak line:**
- Sometimes we want a visual representation of a flow.
- Such a representation is provided by timelines, pathlines, streaklines, and streamlines.
- **Timeline:**
- If a number of adjacent fluid particles in a flow field are marked at a given instant, they form a line in the fluid at that instant; this line is called a **timeline**.
- Subsequent observations of the line may provide information about the flow field.
- For example, in discussing the behavior of a fluid under the action of a constant shear force, timelines were introduced to demonstrate the deformation of a fluid at successive instants.



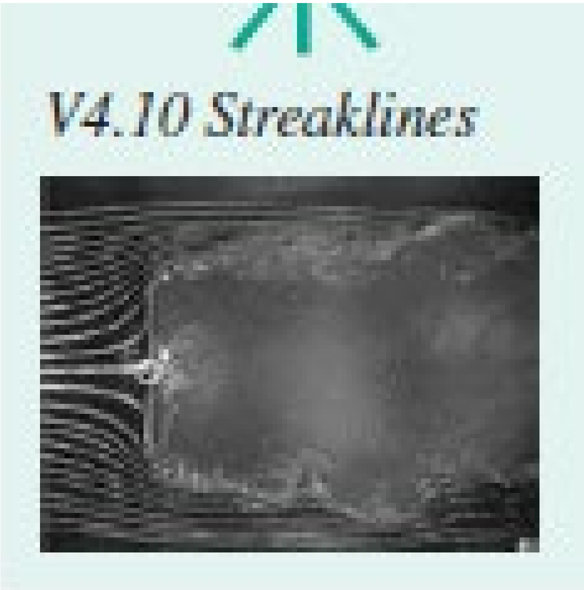

(b) Fluid

- **Flow Visualizaion – Stream line, Path line and Streak line:**
- **Pathline:**
- Pathline is the path or trajectory traced out by a moving fluid particle.
- To make a pathline visible, we might identify a fluid particle at a given instant, e.g., by the use of dye or smoke, and then take a long exposure photograph of its subsequent motion.
- The line traced out by the particle is a pathline.
- This approach might be used to study, for example, the trajectory of a contaminant leaving a smokestack.

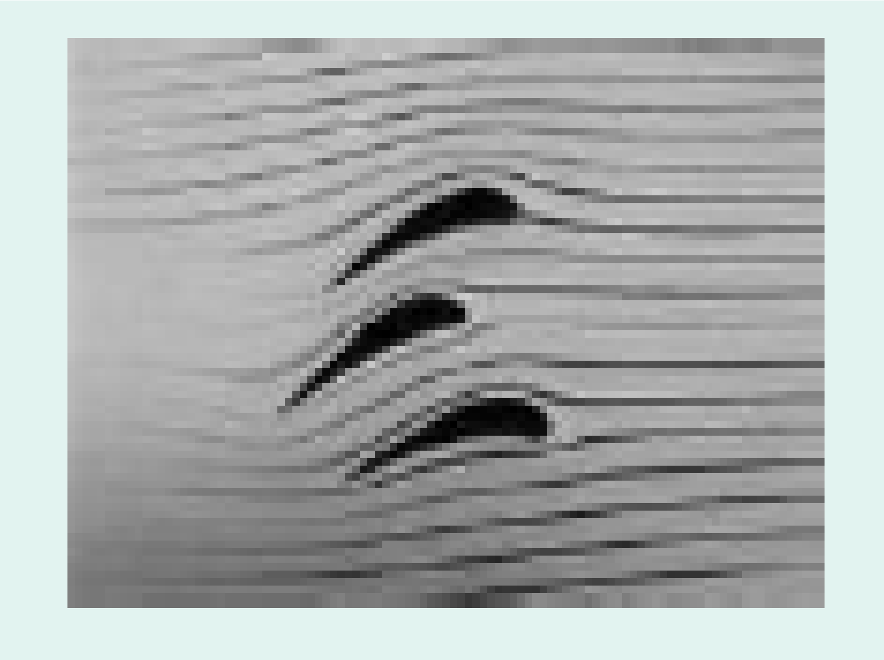


- **Flow Visualizaion – Stream line, Path line and Streak line:**
- **Streakline:**
- On the other hand, we might choose to focus our attention on a fixed location in space and identify, again by the use of dye or smoke, all fluid particles passing through this point.
- After a short period of time we would have a number of identifiable fluid particles in the flow, all of which had, at some time, passed through one fixed location in space.
- The line joining these fluid particles is defined as a **streakline**.



- **Flow Visualizaion – Stream line, Path line and Streak line:**
- **Streamline:**
- Streamlines are lines drawn in the flow field so that at a given instant they are tangent to the direction of flow at every point in the flow field.
- Since the streamlines are tangent to the velocity vector at every point in the flow field, there can be no flow across a streamline.
- Streamlines are the most commonly used visualization technique.



- **Flow Visualizaion – Stream line, Path line and Streak line:**
- In steady flow, the velocity at each point in the flow field remains constant with time and, consequently, the streamline shapes do not vary from one instant to the next.
- This implies that a particle located on a given streamline will always move along the same streamline.
- Furthermore, consecutive particles passing through a fixed point in space will be on the same streamline and, subsequently, will remain on this streamline.
- Thus in a steady flow, pathlines, streaklines, and streamlines are identical lines in the flow field.

- The shapes of the streamlines may vary from instant to instant if the flow is unsteady.
- In the case of unsteady flow, pathlines, streaklines, and streamlines do not coincide.

- **Viscosity:**
- For a solid, stresses develop when the material is elastically deformed or strained; for a fluid, shear stresses arise due to viscous flow.
- Hence we say solids are elastic, and fluids are viscous.
- As two fluids (such as water and oil) can have approximately the same value of density but behave quite differently when flowing.
- There is apparently some additional property that is needed to describe the "fluidity" of the fluid (i.e., how easily it flows).
- The most important of these is **viscosity**, which relates the local stresses in a moving fluid to the strain rate of the fluid element.
- Viscosity is a quantitative measure of a fluid's resistance to flow.
- More specifically, it determines the fluid strain rate that is generated by a given applied shear stress.
- We can easily move through air, which has very low viscosity.
- Movement is more difficult in water, which has 50 times higher viscosity.
- Still more resistance is found in SAE 30 oil, which is 300 times more viscous than water.
- Fluids may have a vast range of viscosities.

- **Viscosity:**
- To determine this additional property, consider a hypothetical experiment in which a material is placed between two very wide parallel plates as shown in Fig.
- The bottom plate is rigidly fixed, but the upper plate is free to move.
- When the constant force δF is applied to the upper plate, it will move continuously with a constant velocity δU.
- This behavior is consistent with the definition of a fluid—that is, if a shearing stress is applied to a fluid it will deform continuously.
- A closer inspection of the fluid motion between the two plates would reveal that the fluid in contact with the upper plate moves with the plate velocity, δU, and the fluid in contact with the bottom fixed plate has a zero velocity.
- The experimental observation that the fluid "sticks" to the solid boundaries is a very important one in fluid mechanics and is usually referred to as the no-slip condition.
- All fluids, both liquids and gases, satisfy this condition.

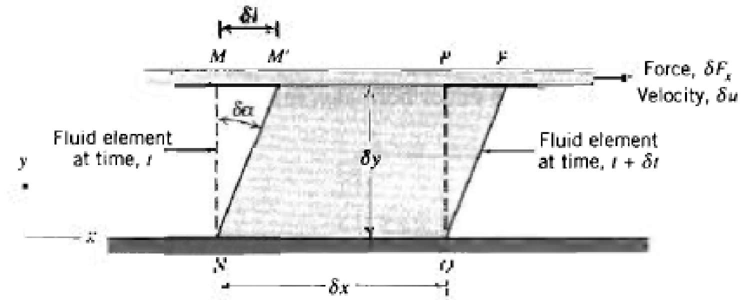


Fig. 2.7 Deformation of a fluid element.

- **Viscosity:**
- The shear stress, $\zeta_{YX}$ , applied to the fluid element is given by

$$\tau_{yx} = \lim_{\delta A_y \to 0} \frac{\delta F_x}{\delta A_y} = \frac{dF_x}{dA_y}$$

- where $\delta A_y$ is the area of contact of a fluid element with the plate, and $\delta F_X$ is the force exerted by the plate on that element.
- During time interval δt, the fluid element is deformed from position MNOP to position M'NOP'.
- The rate of deformation or rate of shear strain of the fluid is given by:

$$deformation\ rate = \lim_{\delta t \to 0} \frac{\delta \alpha}{\delta t} = \frac{d\alpha}{dt}$$

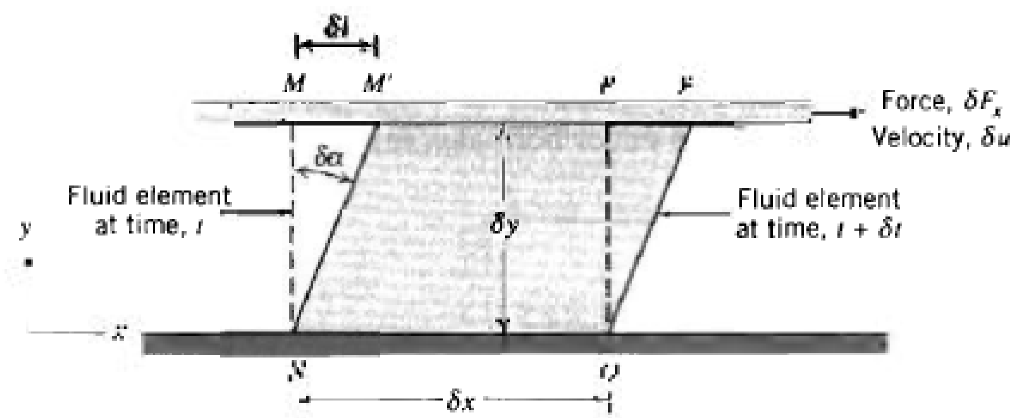



Fig. 2.7 Deformation of a fluid element.

- **Viscosity:**
- The distance, $\delta l$, between the points M and M' is given by

$$\delta l = \delta u \, \delta t$$

- For small angles; $\tan \delta\alpha = \delta l / \delta y$

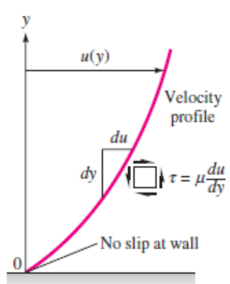
$$\delta l = \delta y \, \delta\alpha$$

- Now, $\delta u \, \delta t = \delta y \, \delta\alpha$
- Rearrange the terms

$$\frac{\delta\alpha}{\delta t} = \frac{\delta u}{\delta y}$$

- Taking the limits of both sides of the equality, we obtain

$$\frac{d\alpha}{dt} = \frac{du}{dy}$$



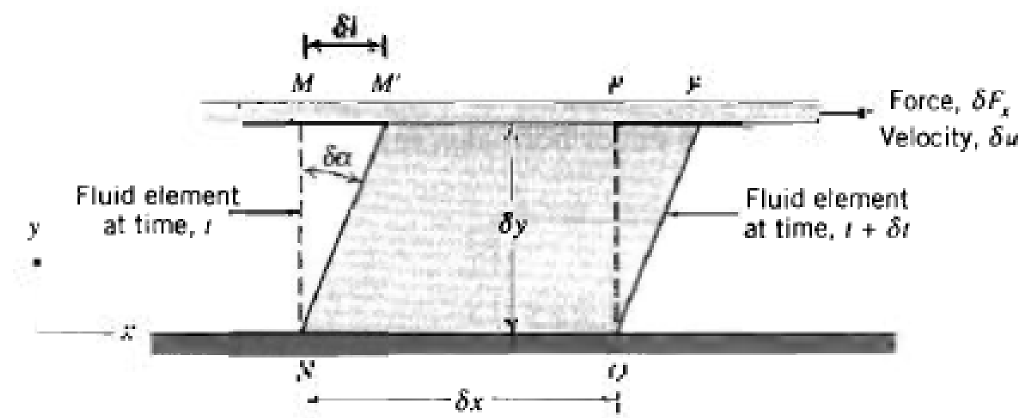




Fig. 2.7 Deformation of a fluid element.

- **Viscosity:**
- Thus, the fluid element, when subjected to shear stress, $\zeta_{yx}$, experiences a rate of deformation {shear rate) given by du/dy.
- The rate of shearing strain is increased in direct proportion—that is,

$$\tau \propto \frac{du}{dy}$$

- This result indicates that for common fluids, such as water, oil, gasoline, and air, the shearing stress and rate of shearing strain (velocity gradient) can be related with a relationship of the form:

$$\tau = \mu \frac{du}{dy}$$

- where the constant of proportionality is designated by the Greek symbol μ (mu) and is called the absolute viscosity, dynamic viscosity, or simply the viscosity of the fluid.

- **Viscosity:**
- Kinematic viscosity is defined as the ratio between dynamic viscosity and density of fluid.
- Thus, we have:

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho}$$

- **Viscosity:**

**Problem 1.18** *Two large plane surfaces are 2.4 cm apart. The space between the surfaces is filled with glycerine. What force is required to drag a very thin plate of surface area 0.5 square metre between the two large plane surfaces at a speed of 0.6 m/s, if :*

*(i) the thin plate is in the middle of the two plane surfaces, and*

*(ii) the thin plate is at a distance of 0.8 cm from one of the plane surfaces ? Take the dynamic viscosity of glycerine =* $8.10 \times 10^{-1}$ *N s/m*$^2$.

**Solution.** Given :

Distance between two large surfaces = 2.4 cm

Area of thin plate, $A = 0.5 \text{ m}^2$

Velocity of thin plate, $u = 0.6$ m/s

Viscosity of glycerine, $\mu = 8.10 \times 10^{-1}$ N s/m$^2$

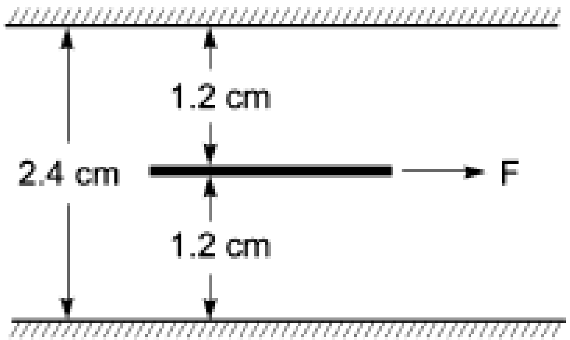


Fig. 1.7 (*a*)

**Case I.** When the thin plate is in the middle of the two plane surfaces [Refer to Fig. 1.7 (*a*)]

Let $F_1$ = Shear force on the upper side of the thin plate

$F_2$ = Shear force on the lower side of the thin plate

$F$ = Total force required to drag the plate

Then $F = F_1 + F_2$

The shear stress ($\tau_1$) on the upper side of the thin plate is given by equation,

- **Viscosity:**

$$\tau_1 = \mu \left(\frac{du}{dy}\right)_1$$

where $du$ = Relative velocity between thin plate and upper large plane surface
= 0.6 m/sec

$dy$ = Distance between thin plate and upper large plane surface
= 1.2 cm = 0.012 m (plate is a thin one and hence thickness of plate is neglected)

$\therefore$ $$\tau_1 = 8.10 \times 10^{-1} \times \left(\frac{0.6}{.012}\right) = 40.5 \text{ N/m}^2$$

Now shear force, $F_1$ = Shear stress × Area

$= \tau_1 \times A = 40.5 \times 0.5 = 20.25$ N

Similarly shear stress ($\tau_2$) on the lower side of the thin plate is given by

$$\tau_2 = \mu \left(\frac{du}{dy}\right)_2 = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.012}\right) = 40.5 \text{ N/m}^2$$

$\therefore$ Shear force, $F_2 = \tau_2 \times A = 40.5 \times 0.5 = 20.25$ N

$\therefore$ Total force, $F = F_1 + F_2 = 20.25 + 20.25 =$ **40.5 N. Ans.**

- **Viscosity:**

**Case II.** When the thin plate is at a distance of 0.8 cm from one of the plane surfaces [Refer to Fig. 1.7 (*b*)].

Let the thin plate is at a distance 0.8 cm from the lower plane surface.

Then distance of the plate from the upper plane surface

$$= 2.4 - 0.8 = 1.6 \text{ cm} = .016 \text{ m}$$

(Neglecting thickness of the plate)

The shear force on the upper side of the thin plate,

$$F_1 = \text{Shear stress} \times \text{Area} = \tau_1 \times A$$

$$= \mu \left(\frac{du}{dy}\right)_1 \times A = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.016}\right) \times 0.5 = 15.18 \text{ N}$$

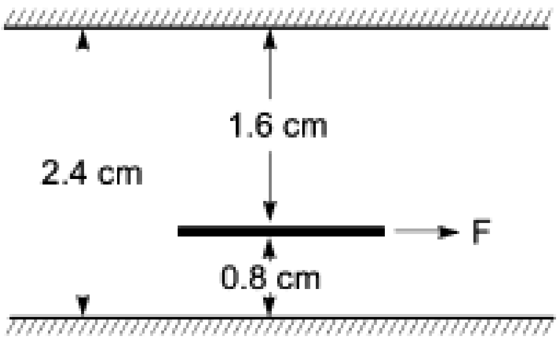



Fig. 1.7 (*b*)

The shear force on the lower side of the thin plate,

$$F_2 = \tau_2 \times A = \mu \left(\frac{du}{dy}\right)_2 \times A$$

$$= 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.8/100}\right) \times 0.5 = 30.36 \text{ N}$$

$\therefore$ Total force required $= F_1 + F_2 = 15.18 + 30.36 =$ **45.54 N. Ans.**

- **Newtonian Fluid:**
- Most common fluids such as water, air, and gasoline are Newtonian under normal conditions.
- If the fluid is Newtonian, then

$$\tau_{yx} \propto \frac{du}{dy}$$

- It states that the shear stress on a fluid element layer is directly proportional to the rate of shear strain or velocity gradient.
- The constant of proportionality is called the co-efficient of viscosity.

$$\tau = \mu \frac{du}{dy}.$$

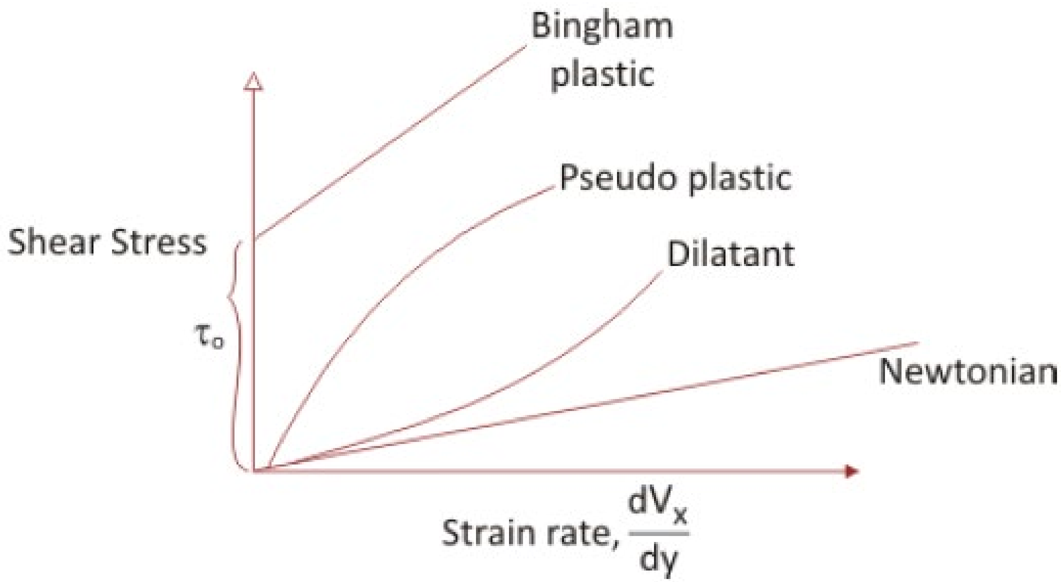
- **Non-Newtonian Fluid:**
- Fluids in which shear stress is not directly proportional to deformation rate are non-Newtonian.
- Non-Newtonian fluids commonly are classified as having time-independent or time-dependent behavior.
- Familiar example is toothpaste.
- Toothpaste behaves as a "fluid" when squeezed from the tube. However, it does not run out by itself when the cap is removed.
- There is a threshold or yield stress below which toothpaste behaves as a solid.
- Strictly speaking, our definition of a fluid is valid only for materials that have zero yield stress.
- This may be adequately represented for many engineering applications by the **power law model**, which for one-dimensional flow becomes

$$\tau_{yx} = k\left(\frac{du}{dy}\right)^n \qquad (2.11)$$

where the exponent, $n$, is called the flow behavior index and the coefficient, $k$, the consistency index. This equation reduces to Newton's law of viscosity for $n = 1$ with $k = \mu$.

- **Newtonian Fluid:**




- **Newtonian:** $\tau = \mu \frac{dV_x}{dy}$: air, water, glycerin
- **Bingham Plastic:** $\tau = \underset{\text{yield stress}}{\tau_o} + \mu \frac{dV_x}{dy}$ : toothpaste

  (Fluid does not move or deform till there is a critical stress)
- **Dilatant:** $\tau = K \left(\frac{dV_x}{dy}\right)^n, n > 1$: starch or sand suspension

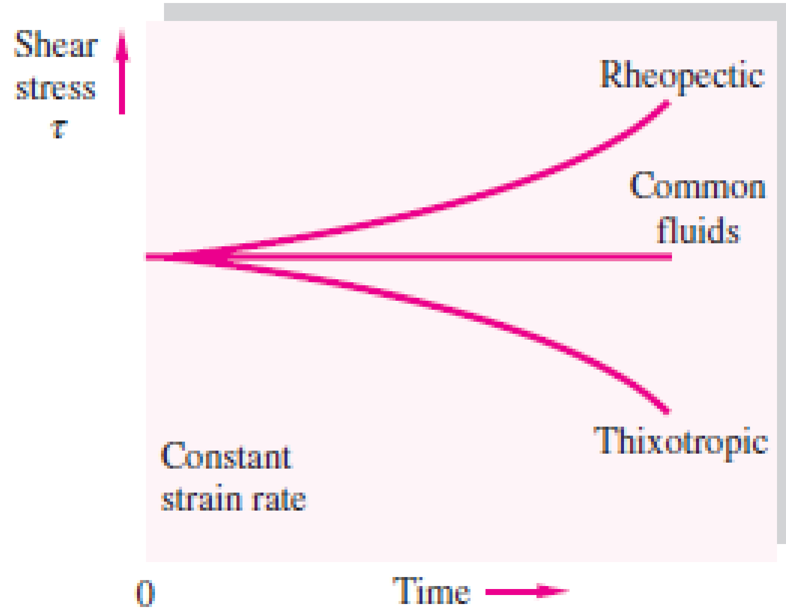
  or shear thickening fluid

  (Fluid starts 'thickening' with increase in its apparent viscosity)
- **Pseudo plastic:** $\tau = K \left(\frac{dV_x}{dy}\right)^n, n < 1$: paint or shear thinning fluid

  (Fluid starts 'thinning' with decrease in its apparent viscosity)

- **Newtonian Fluid:**




A further complication of nonnewtonian behavior is the transient effect shown in Fig. 1.9*b*. Some fluids require a gradually increasing shear stress to maintain a constant strain rate and are called *rheopectic*. The opposite case of a fluid that thins out with time and requires decreasing stress is termed *thixotropic*. We neglect non-

- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**

- **Reference:**
- Fluid Mechanics by Fox
- Fluid Mechanics by Bansal
- Fluid Mechanics by Young
- NPTEL