CL24203 – FLUID MECHANICS

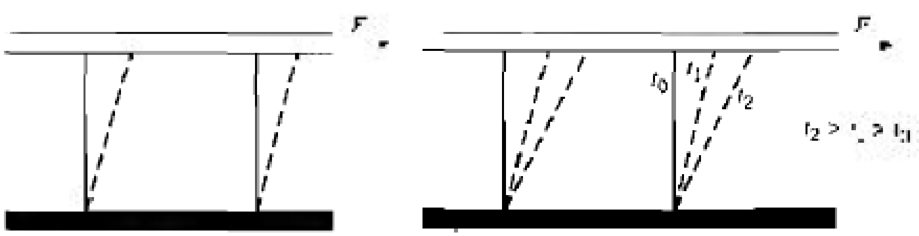
III Semester BTech (Chemical Engineering)

- **Module 1:**
- Fluid Statics:
- Basic equation of fluid statics;
- Pressure variation in a static field;
- Pressure measuring devices–manometer, U-tube, inclined tube, well, diaphragm,
- Hydraulic systems – force on submerged bodies (straight, inclined),
- Pressure centre.

[8]

- Fluid mechanics: Subject that deals with the study of the behavior of a fluid either at rest or in motion.
- Branch of science deals with static, kinematics and dynamic aspects of fluids.
- Study of fluids at rest is called fluid statics.
- Study of fluid in motion, where pressure forces are not considered is called kinematics (often referred as the geometry of motion). It is generally a continuous function in space and time.
- If pressure forces are considered for the fluid in motion is called fluid dynamics.
- Fluids tend to flow when we interact with them whereas solids tend to deform or bend.
- A fluid is a substance that deforms continuously under the application of a shear (tangential) stress no matter how small the shear stress may be.
- Distinction between a fluid and the solid is clear if you compare fluid and solid behavior.
- A solid deforms when a shear stress is applied, but its deformation does not continue to increase with time.

- The shear force is applied to the solid through the upper of two plates to which the solid has been bonded.
- When the shear force is applied to the plate, the block is deformed as in the Figure.
- The top layer is now stationary. The solid has, therefore, resisted the applied shear force.
- Provided the elastic limit of the solid material is not exceeded, the deformation is proportional to the applied shear stress.
- For Fluids, When the shear force, F, is applied to the upper plate, the deformation of the fluid element continues to increase as long as the force is applied.
- The shape of the fluid element, at successive instants of time t2> t1> t0, is shown by the dashed lines. **Conclusion:**
- Fluid continues to deform (or move) under the application of a shear force.
- Fluid at rest cannot sustain a shear stress.



(a) Solid (b) Fluid

- # CL203 FLUID MECHANICS

Distinction Between Solid and Fluid

Solid	Fluid
▪ More Compact Structure ▪ Attractive Forces between the molecules are larger therefore more closely packed ▪ Solids can resist tangential stresses in static condition ▪ Whenever a solid is subjected to shear stress a. It undergoes a definite deformation α or breaks b. α is proportional to shear stress upto some limiting condition ▪ Solid may regain partly or fully its original shape when the tangential stress is removed	▪ Less Compact Structure ▪ Attractive Forces between the molecules are smaller therefore more loosely packed ▪ Fluids cannot resist tangential stresses in static condition. ▪ Whenever a fluid is subjected to shear stress a. No fixed deformation b. Continious deformation takes place until the shear stress is applied ▪ A fluid can never regain its original shape, once it has been distorded by the shear stress

- Source: NPTEL Lecture

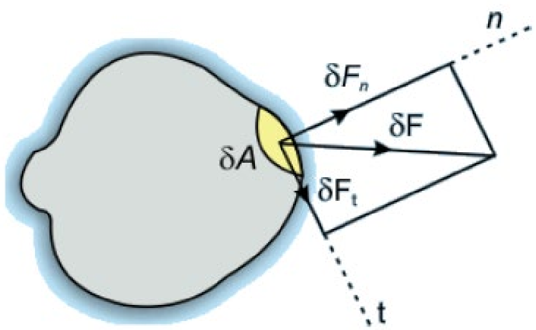
- **Definition of Stress:**
- Consider a small area δA on the surface of a body.
- The force acting on this area is δF.
- This force can be resolved into two perpendicular components.
- The component of force acting normal to the area called normal force and is denoted by δFn
- The component of force acting along the plane of area is called tangential force and is denoted by δFt.
- When they are expressed as force per unit area they are called as normal stress and tangential stress respectively.
- The tangential stress is also called shear stress.

The normal stress

$$\sigma = \lim_{\delta A \to 0}\left(\frac{\delta F_n}{\delta A}\right)$$

And shear stress

$$\tau = \lim_{\delta A \to 0}\left(\frac{\delta F_t}{\delta A}\right)$$



- **CL203 FLUID MECHANICS**
- **PROPERTIES OF FLUID:**
- **Density:** It is defined as the ratio of mass of fluid to its volume.

Mathematically, mass density is written as

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}.$$

The value of density of water is 1 gm/cm^3 or 1000 kg/m^3.

- **Specific weight or weight Density**: It is the ratio of weight of fluid to its volume

Thus mathematically,

$$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{(\text{Mass of fluid}) \times \text{Acceleration due to gravity}}{\text{Volume of fluid}}$$

$$= \frac{\text{Mass of fluid} \times g}{\text{Volume of fluid}}$$

$$= \rho \times g \qquad \left\{\because \frac{\text{Mass of fluid}}{\text{Volume of fluid}} = \rho\right\}$$

$$\therefore \quad w = \rho g \qquad ...(1.1)$$

- **CL203 FLUID MECHANICS**
- **PROPERTIES OF FLUID:**
- **Specific Volume:** It is defined as the ratio of volume of fluid to mass of fluid.

$$\text{Specific volume} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\frac{\text{Mass of fluid}}{\text{Volume of fluid}}} = \frac{1}{\rho}$$

Thus specific volume is the reciprocal of mass density. It is expressed as m^3/kg. It is commonly applied to gases.

- **Specific Gravity**: It is the ratio of weight density of fluid to the weight density fo a standard fluid.

$$\text{Mathematically,}\ \ S(\text{for liquids}) = \frac{\text{Weight density (density) of liquid}}{\text{Weight density (density) of water}}$$

- **CL203 FLUID MECHANICS**
- **PROPERTIES OF FLUID:**
- **Viscosity:** It is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of fluid.

Mathematically, $$\tau \propto \frac{du}{dy}$$

or $$\tau = \mu \frac{du}{dy}$$

- Where mu is the constant of proportionality and is known as coefficient of dynamic viscosity. du/dy is rate of shear strain or velocity gradient.
- Thus viscosity is defined as the shear stress required to produce unit rate of shear strain.

- **CL203 FLUID MECHANICS**
- **PROPERTIES OF FLUID:**
- **Unit of Viscosity:**

$$\mu = \frac{\text{Shear stress}}{\dfrac{\text{Change of velocity}}{\text{Change of distance}}} = \frac{\text{Force/Area}}{\left(\dfrac{\text{Length}}{\text{Time}}\right) \times \dfrac{1}{\text{Length}}}$$

$$= \frac{\text{Force/(Length)}^2}{\dfrac{1}{\text{Time}}} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2}$$

$$\text{MKS unit of viscosity} = \frac{\text{kgf-sec}}{\text{m}^2}$$

In the above expression N/m^2 is also known as Pascal which is represented by Pa. Hence N/m^2 = Pa = Pascal

$\therefore$ SI unit of viscosity = Ns/m^2 = Pa s.

$$\text{SI unit of viscosity} = \frac{\text{Newton-sec}}{\text{m}^2} = \frac{\text{Ns}}{\text{m}^2}$$

- **If unit given in poise then,**

$$\text{One poise} = \frac{1}{10} \frac{\text{Ns}}{\text{m}^2}.$$

- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**
- **Introduction:**
- A fluid as a substance that will continuously deform, or flow, whenever a shear stress is applied to it.
- It follows that for a fluid at rest the shear stress must be zero.
- We can conclude that for a static fluid only normal stress is present—in other words, pressure.
- We will study the topic of fluid statics in this module.
- The pressure generated within a static fluid is an important phenomenon in many practical situations.
- Using the principles of hydrostatics, we can compute forces on submerged objects, develop instruments for measuring pressures, and deduce properties of the atmosphere and oceans.
- The principles of hydrostatics also may be used to determine the forces developed by hydraulic systems in applications such as industrial presses or automobile brakes.

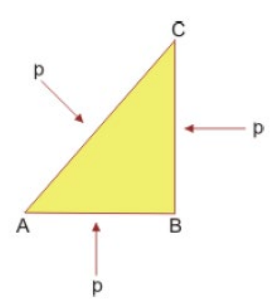
- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**
- **Fluid pressure at a point:**
- Consider a small area dA in large mass of fluid. If the fluid is stationary, then the force exerted by the surrounding fluid on the area dA will always be perpendicular to the surface dA. (In a stationary fluid, shear stress $\zeta = 0$. However, fluid can sustain the normal stress.
- Let dF is the force acting on the area dA in the normal direciton. Then the ratio of dF/dA is known as the intensity of pressure and it is represented by p.

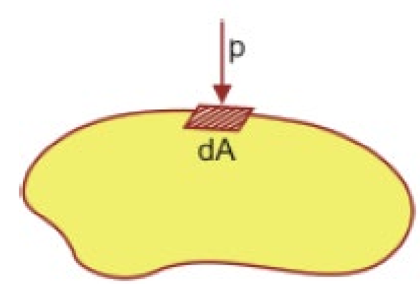
$$p = \frac{dF}{dA}.$$

If the force (F) is uniformly distributed over the area (A), then pressure at any point is given by

$$p = \frac{F}{A} = \frac{\text{Force}}{\text{Area}}.$$

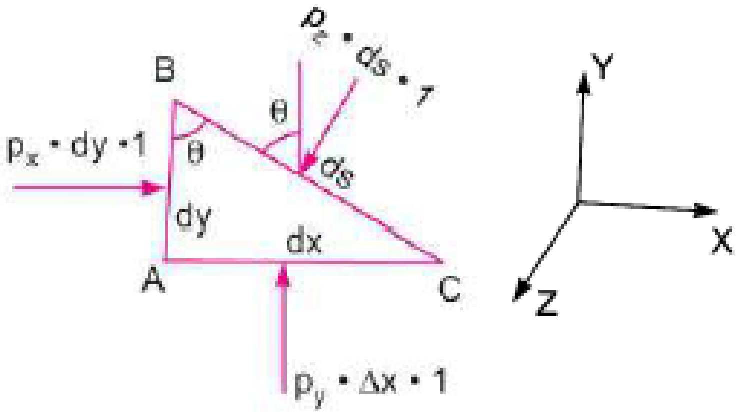
$\therefore$ Force or pressure force, $F = p \times A$.





- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**
- **Fluid elements:**
- Fluid element can be defined as an infinitesimal region of the fluid continuum in isolation from its surroundings.
- Two types of forces exist on fluid elements
- **Body Force:** distributed over the entire mass or volume of the element. It is usually expressed per unit mass of the element or medium upon which the forces act. Example: Gravitational Force, Electromagnetic force fields etc.
- **Surface Force:** Forces exerted on the fluid element by its surroundings through direct contact at the surface.
- Surface force has two components:
- Normal Force: along the normal to the area
- Shear Force: along the plane of the area.

- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**
- **Pascals Law:**
- It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions.
- **Prove:**
- Consider an arbitrary triangular fluid element ABC.
- Pressure acts on all 3-faces of the element inward and normal to the surface with fluid element of very small dimensions i.e., dx, dy and ds.
- Consider the width the element is unity.
- Px, py and pz are the pressures or intensity of pressure acting on the face AB, AC and BC respectively. Let $\angle ABC = \theta$
- Forces acting on the element are:
- 1. Pressure forces normal to the surfaces and
- 2. Weight of element in the vertical direction.
-



- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**
- **Pascals Law:**
- It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions.
- **Prove:**
- The forces on the faces are :

Force on the face AB $= p_x \times$ Area of face AB

$= p_x \times dy \times 1$

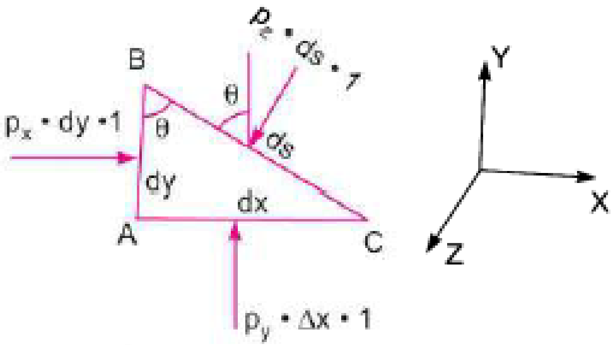
Similarly force on the face AC $= p_y \times dx \times 1$

Force on the face BC $= p_z \times ds \times 1$

Weight of element $=$ (Mass of element) $\times g$

$$= (\text{Volume} \times \rho) \times g = \left(\frac{AB \times AC}{2} \times 1\right) \times \rho \times g,$$

where ρ = density of fluid.



- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**
- **Pascals Law:**
- It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions.
- **Prove:**
- Forces $\sum F = ma = 0$ (fluid is at rest)

Resolving the forces in x-direction, we have

$$p_x \times dy \times 1 - p\,(ds \times 1)\sin(90° - \theta) = 0$$

or $$p_x \times dy \times 1 - p_z\, ds \times 1 \cos\theta = 0.$$

But from Fig. 2.1, $$ds \cos\theta = AB = dy$$

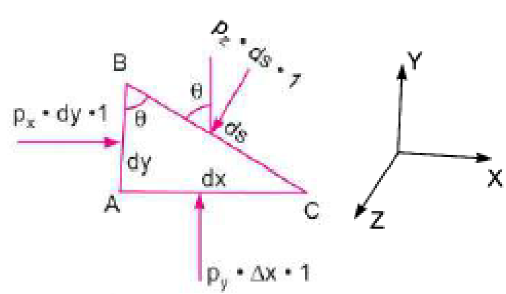
$\therefore$ $$p_x \times dy \times 1 - p_z \times dy \times 1 = 0$$

or $$p_x = p_z \qquad \text{...(2.1)}$$

Similarly, resolving the forces in y-direction, we get

$$p_y \times dx \times 1 - p_z \times ds \times 1 \cos(90° - \theta) - \frac{dx \times dy}{2} \times 1 \times \rho \times g = 0$$

or $$p_y \times dx - p_z\, ds \sin\theta - \frac{dxdy}{2} \times \rho \times g = 0.$$



- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**
- **Pascals Law:**
- It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions.
- **Prove:**

But $ds \sin\theta = dx$ and also the element is very small and hence weight is negligible.

$\therefore$ $$p_y dx - p_z \times dx = 0$$

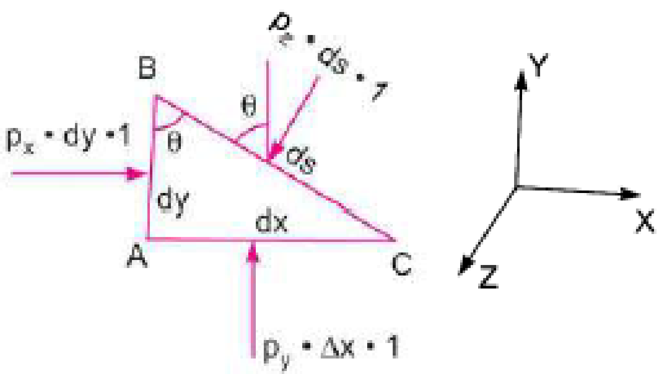
or $$p_y = p_z \quad \text{...(2.2)}$$

From equations (2.1) and (2.2), we have

$$p_x = p_y = p_z \quad \text{...(2.3)}$$

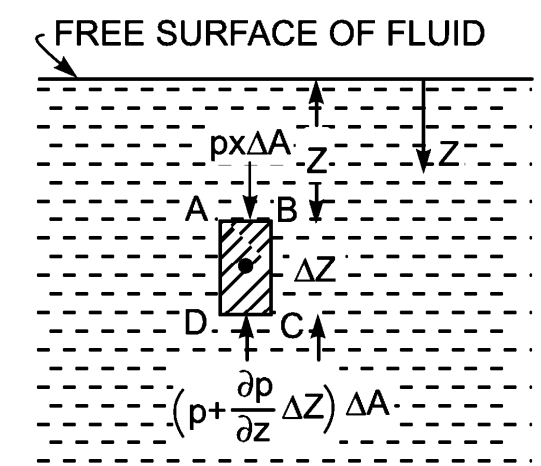
The above equation shows that the pressure at any point in x, y and z directions is equal.

Since the choice of fluid element was completely arbitrary, which means the pressure at any point is the same in all directions.



- **CL203 FLUID MECHANICS**

- **FLUID STATICS:**
- **Pressure variation in a fluid at rest:**
- Pressure at any point in a fluid at rest is obtained by the Hydrostatic Law which states that the rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at that point.
- **Prove:**
- Consider a small fluid element as shown in the figure.
- Let ΔA = Cross-sectional area of element
- ΔZ = Height of fluid element.
- p = Pressure on face AB
- Z = Distance of fluid element from free surface.
- Then, the forces acting on the fluid element are:



2.2 *Forces on a fluid element.*

- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**
- **Pressure variation in a fluid at rest:**
- **Prove:**
- Then, the forces acting on the fluid element are:

1. Pressure force on $AB = p \times \Delta A$ and acting perpendicular to face AB in the downward direction.
2. Pressure force on $CD = \left(p + \frac{\partial p}{\partial Z}\Delta Z\right) \times \Delta A$, acting perpendicular to face CD, vertically upward direction.
3. Weight of fluid element = Density $\times g \times$ Volume = $\rho \times g \times (\Delta A \times \Delta Z)$.

- For equilibrium of fluid element, we have:

$$p\Delta A - \left(p + \frac{\partial p}{\partial Z}\Delta Z\right)\Delta A + \rho \times g \times (\Delta A \times \Delta Z) = 0$$

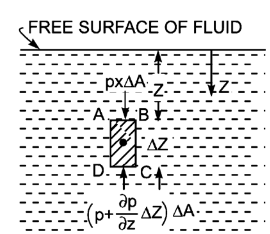
or $$p\Delta A - p\Delta A - \frac{\partial p}{\partial Z}\Delta Z\Delta A + \rho \times g \times \Delta A \times Z = 0$$

or $$-\frac{\partial p}{\partial Z}\Delta Z\Delta A + \rho \times g \times \Delta A\Delta Z = 0$$

or $$\frac{\partial p}{\partial Z}\Delta Z\Delta A = \rho \times g \times \Delta A\Delta Z \quad \text{or} \quad \frac{\partial p}{\partial Z} = \rho \times g \text{ [cancelling } \Delta A\Delta Z \text{ on both sides]}$$

$$\therefore \quad \frac{\partial p}{\partial Z} = \rho \times g = w \qquad (\because \rho \times g = w)$$

where w = Weight density of fluid.



2.2 *Forces on a fluid element.*

- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**
- **Pressure variation in a fluid at rest:**
- **Prove:**
- Then, the forces acting on the fluid element are:

Equation (2.4) states that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point. This is **Hydrostatic Law.**

By integrating the above equation (2.4) for liquids, we get

$$\int dp = \int \rho g dZ$$

or

$$p = \rho g Z \qquad \text{...(2.5)}$$

where p is the pressure above atmospheric pressure and Z is the height of the point from free surfaces.

From equation (2.5), we have

$$Z = \frac{p}{\rho \times g} \qquad \text{...(2.6)}$$

Here Z is called **pressure head.**

- Physical significance of each term by the hydrostatic law:

$$\left(-\nabla p + \rho \vec{g}\right) = 0 \qquad \text{L -7.7}$$

$$\begin{bmatrix}\text{Net pressure force}\\ \text{per unit volume}\\ \text{at a point}\end{bmatrix} + \begin{bmatrix}\text{Body force}\\ \text{per unit volume}\\ \text{at a point}\end{bmatrix} = 0$$

- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**
- **Pressure variation in a fluid at rest:**

Problem 2.1 *A hydraulic press has a ram of 30 cm diameter and a plunger of 4.5 cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 500 N.*

Solution. Given :

Dia. of ram, $D = 30 \text{ cm} = 0.3 \text{ m}$

Dia. of plunger, $d = 4.5 \text{ cm} = 0.045 \text{ m}$

Force on plunger, $F = 500 \text{ N}$

Find weight lifted $= W$

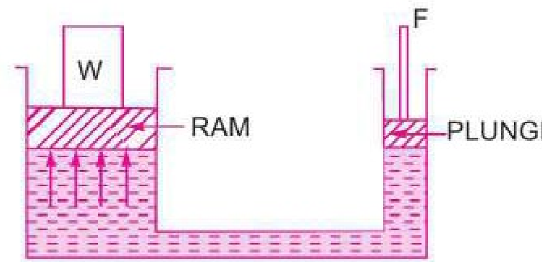
Area of ram, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.3)^2 = 0.07068 \text{ m}^2$

Area of plunger, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.045)^2 = .00159 \text{ m}^2$

Pressure intensity due to plunger

$$= \frac{\text{Force on plunger}}{\text{Area of plunger}} = \frac{F}{a} = \frac{500}{.00159} \text{ N/m}^2.$$

Due to Pascal's law, the intensity of pressure will be equally transmitted in all directions. Hence the pressure intensity at the ram



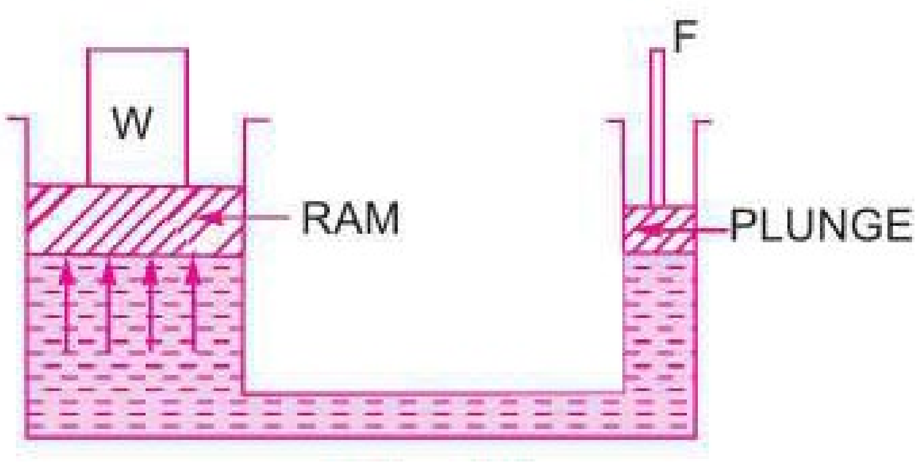
- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**
- **Pressure variation in a fluid at rest:**
- The pressure intensity at ram is:

$$= \frac{500}{.00159} = 314465.4 \text{ N/m}^2$$

But pressure intensity at ram $= \dfrac{\text{Weight}}{\text{Area of ram}} = \dfrac{W}{A} = \dfrac{W}{.07068} \text{ N/m}^2$

$$\frac{W}{.07068} = 314465.4$$

$\therefore$ Weight $= 314465.4 \times .07068 = 22222 \text{ N} =$ **22.222 kN. Ans.**



- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**
- **Pressure variation in a fluid at rest:**

Problem 2.6 *An open tank contains water upto a depth of 2 m and above it an oil of sp. gr. 0.9 for a depth of 1 m. Find the pressure intensity (i) at the interface of the two liquids, and (ii) at the bottom of the tank.*

Solution. Given :

Height of water, $Z_1 = 2$ m

Height of oil, $Z_2 = 1$ m

Sp. gr. of oil, $S_0 = 0.9$

Density of water, $\rho_1 = 1000$ kg/m^3

Density of oil, $\rho_2 =$ Sp. gr. of oil × Density of water $= 0.9 \times 1000 = 900$ kg/m^3

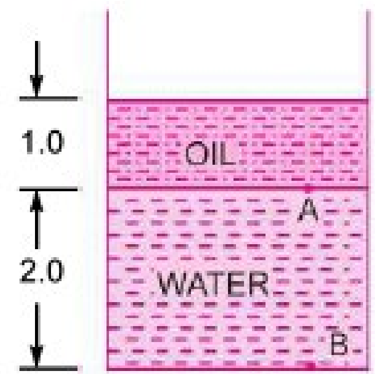


Fig. 2.4

Pressure intensity at any point is given by

$$p = \rho \times g \times Z.$$

(*i*) At interface, *i.e.*, at *A*

$$p = \rho_2 \times g \times 1.0$$
$$= 900 \times 9.81 \times 1.0$$
$$= 8829 \frac{N}{m^2} = \frac{8829}{10^4} = \mathbf{0.8829\ N/cm^2.\ Ans.}$$

- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**
- **Pressure variation in a fluid at rest:**

Problem 2.6 *An open tank contains water upto a depth of 2 m and above it an oil of sp. gr. 0.9 for a depth of 1 m. Find the pressure intensity (i) at the interface of the two liquids, and (ii) at the bottom of the tank.*

Solution. Given :

Height of water, $Z_1 = 2$ m

Height of oil, $Z_2 = 1$ m

Sp. gr. of oil, $S_0 = 0.9$

Density of water, $\rho_1 = 1000$ kg/m^3

Density of oil, ρ_2 = Sp. gr. of oil × Density of water $= 0.9 \times 1000 = 900$ kg/m^3

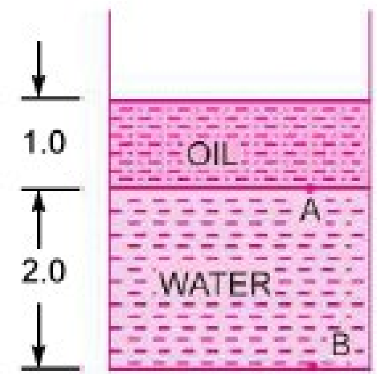


Fig. 2.4

Pressure intensity at any point is given by

$$p = \rho \times g \times Z.$$

(*ii*) At the bottom, *i.e.*, at *B*

$$p = \rho_2 \times gZ_2 + \rho_1 \times g \times Z_1 = 900 \times 9.81 \times 1.0 + 1000 \times 9.81 \times 2.0$$

$$= 8829 + 19620 = 28449 \text{ N/m}^2 = \frac{28449}{10^4} \text{ N/cm}^2 = \mathbf{2.8449\ N/cm^2.\ Ans.}$$

- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**
- **Pressure variation in a fluid at rest:**

Problem 2.7 *The diameters of a small piston and a large piston of a hydraulic jack are 3 cm and 10 cm respectively. A force of 80 N is applied on the small piston. Find the load lifted by the large piston when :*

(a) the pistons are at the same level.

(b) small piston is 40 cm above the large piston.

The density of the liquid in the jack is given as 1000 kg/m^3.

Solution. Given :

Dia. of small piston, $d = 3$ cm

$\therefore$ Area of small piston, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (3)^2 = 7.068 \text{ cm}^2$

Dia. of large piston, $D = 10$ cm

$\therefore$ Area of larger piston, $A = \frac{p}{4} \times (10)^2 = 78.54 \text{ cm}^2$

Force on small piston, $F = 80$ N

Let the load lifted $= W$.

- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**
- **Pressure variation in a fluid at rest:**

(*a*) When the pistons are at the same level

Pressure intensity on small piston

$$\frac{F}{a} = \frac{80}{7.068} \text{ N/cm}^2$$

This is transmitted equally on the large piston.

∴ Pressure intensity on the large piston

$$= \frac{80}{7.068}$$

∴ Force on the large piston = Pressure × Area

$$= \frac{80}{7.068} \times 78.54 \text{ N} = \mathbf{888.96\ N.\ Ans.}$$

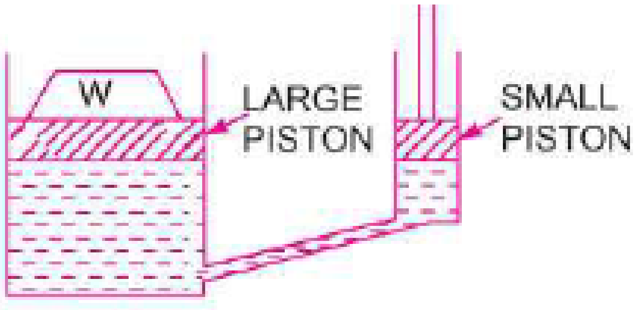


Fig. 2.5

- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**
- **Pressure variation in a fluid at rest:**

(*b*) **When the small piston is 40 cm above the large piston**

Pressure intensity on the small piston

$$= \frac{F}{a} = \frac{80}{7.068} \frac{\text{N}}{\text{cm}^2}$$

∴ Pressure intensity at section *A-A*

$$= \frac{F}{a} + \text{Pressure intensity due to height of 40 cm of liquid.}$$

But pressure intensity due to 40 cm of liquid

$$= \rho \times g \times h = 1000 \times 9.81 \times 0.4 \text{ N/m}^2$$

$$= \frac{1000 \times 9.81 \times .40}{10^4} \text{ N/cm}^2 = 0.3924 \text{ N/cm}^2$$

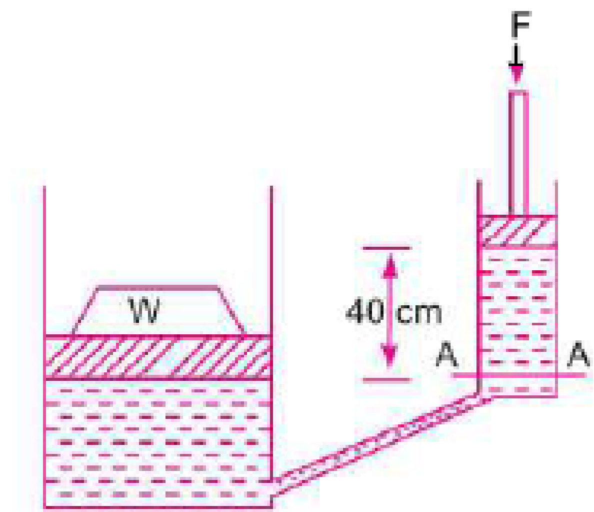


Fig. 2.6

∴ Pressure intensity at section *A-A*

$$= \frac{80}{7.068} + 0.3924$$

$$= 11.32 + 0.3924 = 11.71 \text{ N/cm}^2$$

∴ Pressure intensity transmitted to the large piston = 11.71 N/cm^2

∴ Force on the large piston = Pressure × Area of the large piston

$$= 11.71 \times A = 11.71 \times 78.54 = 919.7 \text{ N.}$$

- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**

- **Reference:**
- Fluid Mechanics by Fox
- Fluid Mechanics by Bansal
- NPTEL

CL24203 – FLUID MECHANICS

III Semester BTech (Chemical Engineering)

CL203 FLUID MECHANICS

- **Module 1:**
- Fluid Statics:
- Basic equation of fluid statics;
- Pressure variation in a static field;
- Pressure measuring devices–manometer, U-tube, inclined tube, well, diaphragm,
- Hydraulic systems – force on submerged bodies (straight, inclined),
- Pressure centre.
  [8]
-

- **Pressure at a point in incompressible fluid:**
- The density of all fluids depends on the temp., and pr., the variation in density with changes in temp., and pr., may be small or large.
- If density changes only slightly with moderate changes in temp., and pr.,, the fluid is said to be incompressible fluid.
- While the density changes are significant, then the fluid is said to be compressible fluid.
- For an incompressible fluid, the density ρ is constant throughout. Hence the Hydrostatic Eq. .can be integrated and expressed.

$$\frac{dp}{dz} = -\rho g$$

- To determine the pressure variation, we must integrate and apply appropriate boundary conditions.
- If the pressure at the reference level, Z0, is designated as p 0 , then the pressure, p, at level z is found by integration:

$$\int_{p_0}^{p} dp = -\int_{z_0}^{z} \rho g \, dz$$

$$p - p_0 = -\rho g(z - z_0) = \rho g(z_0 - z)$$

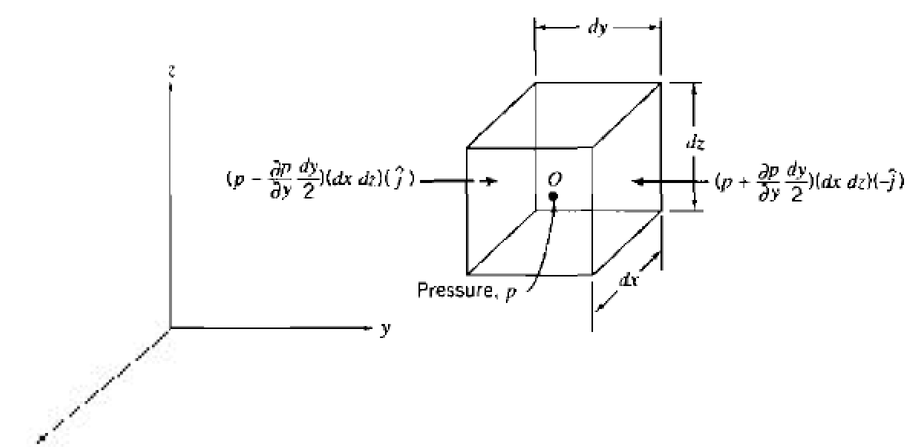
- **Pressure at a point in incompressible fluid:**

$$\frac{dp}{dz} = -\rho g$$

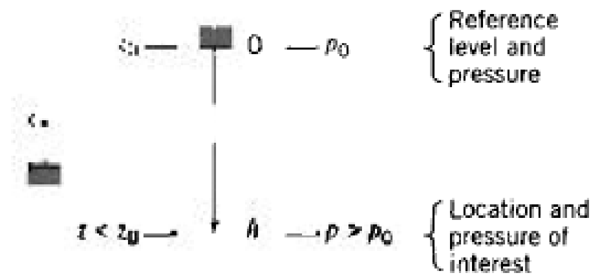
Equations 3.4 describe the pressure variation in each of the three coordinate directions in a static fluid. To simplify further, it is logical to choose a coordinate system such that the gravity vector is aligned with one of the coordinate axes. If the coordinate system is chosen with the z axis directed vertically upward, as in Fig. 3.1, then $g_x = 0$, $g_y = 0$, and $g_z = -g$. Under these conditions, the component equations become

$$\frac{\partial p}{\partial x} = 0 \quad \frac{\partial p}{\partial y} = 0 \quad \frac{\partial p}{\partial z} = -\rho g$$

$$\frac{dp}{dz} = -\rho g \equiv -\gamma$$



- **Pressure at a point in incompressible fluid:**
- For liquids, it is often convenient to take the origin of the coordinate system at the free surface (reference level) and to measure distances as positive downward from the free surface as in Fig.



With h measured positive downward, we have

$$z_0 - z = h$$

$$p - p_0 = \rho g h$$

- Therefore, Eq. gives the expression of hydrostatic pressure p at a point whose vertical depression from the free surface is h.

Similarly,

$$p_1 - p_2 = \rho g (z_1 - z_2) = \rho g h$$

- Thus, the difference in pressure between two points in an incompressible fluid at rest can be expressed in terms of the vertical distance between the points.
- This result is known as Torricelli's principle, which is the basis for differential pressure measuring` devices.
- The pressure p0 at free surface is the local atmospheric pressure.
- Therefore, it can be stated from Eq. that the pressure at any point in an expanse of a fluid at rest, with a free surface exceeds that of the local atmosphere by an amount ρgh, where h is the vertical depth of the point from the free surface.

- **Pressure at a point in compressible fluid:**
- For Compressible fluid, density changes with change in temp., and pr.,.
- The equation of state is in terms of density.

$$\frac{p}{\rho} = RT$$

or $$\rho = \frac{p}{RT}$$

Now equation (2.4) is $$\frac{dp}{dZ} = w = \rho g = \frac{p}{RT} \times g$$

$\therefore$ $$\frac{dp}{p} = \frac{g}{RT} dZ$$

In equation (2.4), Z is measured vertically downward. But if Z is measured vertically up, then $\frac{dp}{dZ} = -\rho g$ and hence equation (2.16) becomes

$$\frac{dp}{p} = \frac{-g}{RT} dZ \qquad ...(2.17)$$

- If temp., T is constant which is true for isothermal process, then the equation can be integrated as:

$$\int_{p_0}^{p} \frac{dp}{p} = -\int_{Z_0}^{Z} \frac{g}{RT} dz = -\frac{g}{RT} \int_{Z_0}^{Z} dz$$

$$\log \frac{p}{p_0} = \frac{-g}{RT} [Z - Z_0]$$

- **Pressure at a point in compressible fluid:**
- For Compressible fluid, density changes with change in temp., and pr.,.

$$\int_{p_0}^{p} \frac{dp}{p} = -\int_{Z_0}^{Z} \frac{g}{RT} dz = -\frac{g}{RT} \int_{Z_0}^{Z} dz$$

$$\log \frac{p}{p_0} = \frac{-g}{RT} [Z - Z_0]$$

where p_0 is the pressure where height is Z_0. If the datum line is taken at Z_0, then $Z_0 = 0$ and p_0 becomes the pressure at datum line.

$$\therefore \quad \log \frac{p}{p_0} = \frac{-g}{RT} Z$$

$$\frac{p}{p_0} = e^{-gZ/RT}$$

or pressure at a height Z is given by $p = p_0 e^{-gZ/RT}$ ...(2.18)

- **Pressure at a point in compressible fluid:**
- For Compressible fluid, density changes with change in temp., and pr.,.

Problem 2.22 (SI Units) *If the atmosphere pressure at sea level is 10.143 N/cm², determine the pressure at a height of 2500 m assuming the pressure variation follows (i) Hydrostatic law, and (ii) isothermal law. The density of air is given as 1.208 kg/m³.*

Solution. Given :

Pressure at sea-level, $p_0 = 10.143 \text{ N/cm}^2 = 10.143 \times 10^4 \text{ N/m}^2$

Height, $Z = 2500$ m

Density of air, $\rho_0 = 1.208 \text{ kg/m}^3$

(*i*) **Pressure by hydrostatic law.** For hydrostatic law, ρ is assumed constant and hence p is given by equation $\frac{dp}{dZ} = -\rho g$

Integrating, we get $\int_{p_0}^{p} dp = \int -\rho g \, dZ = -\rho g \int_{Z_0}^{Z} dZ$

or $p - p_0 = -\rho g [Z - Z_0]$

For datum line at sea-level, $Z_0 = 0$

$\therefore$ $p - p_0 = -\rho g Z$ or $p = p_0 - \rho g Z$

$= 10.143 \times 10^4 - 1.208 \times 9.81 \times 2500$ [$\because$ $\rho = \rho_0 = 1.208$]

$= 101430 - 29626 = 71804 \frac{\text{N}}{\text{m}^2}$ or $\frac{71804}{10^4}$ N/cm²

$= \mathbf{7.18 \text{ N/cm}^2}$. **Ans.**

- **Pressure at a point in compressible fluid:**
- For Compressible fluid, density changes with change in temp., and pr.,.

Problem 2.22 (SI Units) *If the atmosphere pressure at sea level is 10.143 N/cm², determine the pressure at a height of 2500 m assuming the pressure variation follows (i) Hydrostatic law, and (ii) isothermal law. The density of air is given as 1.208 kg/m³.*

Solution. Given :

Pressure at sea-level, $p_0 = 10.143 \text{ N/cm}^2 = 10.143 \times 10^4 \text{ N/m}^2$

Height, $Z = 2500$ m

Density of air, $\rho_0 = 1.208 \text{ kg/m}^3$

(*ii*) **Pressure by Isothermal Law.** Pressure at any height Z by isothermal law is given by equation (2.18) as

$$p = p_0\, e^{-gZ/RT}$$

$$= 10.143 \times 10^4\, e^{-\frac{gZ \times \rho_0}{p_0}} \qquad \left[\because \frac{p_0}{\rho_0} = RT \text{ and } \rho_0 g = w_0\right]$$

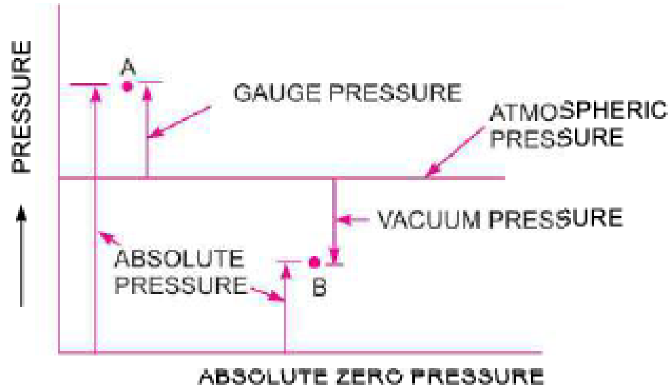
$$= 10.143 \times 10^4\, e^{-\frac{Z\rho_0 \times g}{p_0}}$$

$$= 10.143 \times 10^4\, e^{(-2500 \times 1.208 \times 9.81)/10.143 \times 10^4}$$

$$= 101430 \times e^{-.292} = 101430 \times \frac{1}{1.3391} = 75743 \text{ N/m}^2$$

$$= \frac{75743}{10^4} \text{ N/cm}^2 = \mathbf{7.574\ N/cm^2}. \textbf{ Ans.}$$

- **Absolute, Gauge, Atmospheric and Vacuum Pressures:**
- Pressure on a fluid is measured in two different ways.
- One, it is measured above the absolute zero or complete vacuum and it is called absolute pressure
- In other, pressure is measured above the atmospheric pressure and it is called gauge pressure.
- **1. Absolute Pressure:** It is defined as the pressure which is measured with reference to absolute vacuum pressure.
- **2. Gauge Pressure:** It is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.
- **3. Vacuum Pressure:** It is defined as the pressure below the atmospheric pressure.
- If $p < p_{atm}$, then the gauge pressure becomes negative and is called vacuum pressure.



- **Absolute, Gauge, Atmospheric and Vacuum Pressures:**
- Pressure on a fluid is measured in two different ways.
- Relationship between the absolute pressure, gauge pressure and vacuum pressure are shown in the figure.

Mathematically :

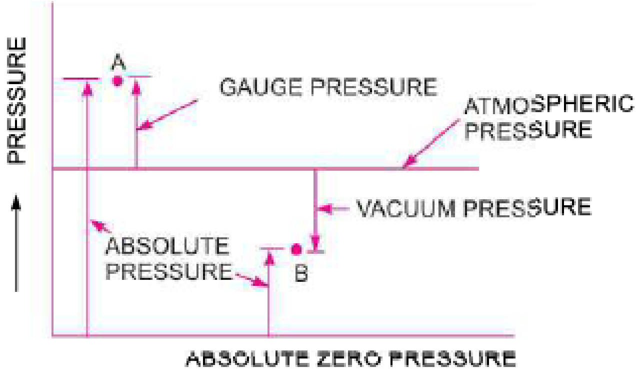
(*i*) Absolute pressure

= Atmospheric pressure + Gauge pressure

or $$p_{ab} = p_{atm} + p_{gauge}$$

(*ii*) Vacuum pressure

= Atmospheric pressure – Absolute pressure.



Note. (*i*) The atmospheric pressure at sea level at 15°C is 101.3 kN/m^2 or 10.13 N/cm^2 in SI unit. In case of MKS units, it is equal to 1.033 kgf/cm^2.

(*ii*) The atmospheric pressure head is 760 mm of mercury or 10.33 m of water.

• Absolute, Gauge, Atmospheric and Vacuum Pressures:

Problem 2.8 *What are the gauge pressure and absolute pressure at a point 3 m below the free surface of a liquid having a density of 1.53×10^3 kg/m^3 if the atmospheric pressure is equivalent to 750 mm of mercury ? The specific gravity of mercury is 13.6 and density of water = 1000 kg/m^3.*

Solution. Given :

Depth of liquid, $Z_1 = 3$ m

Density of liquid, $\rho_1 = 1.53 \times 10^3$ kg/m^3

Atmospheric pressure head, $Z_0 = 750$ mm of Hg

$$= \frac{750}{1000} = 0.75 \text{ m of Hg}$$

∴ Atmospheric pressure, $p_{atm} = \rho_0 \times g \times Z_0$

where ρ_0 = Density of Hg = Sp. gr. of mercury × Density of water = 13.6×1000 kg/m^3

and Z_0 = Pressure head in terms of mercury.

$$\therefore \quad p_{atm} = (13.6 \times 1000) \times 9.81 \times 0.75 \text{ N/m}^2 \qquad (\because \quad Z_0 = 0.75)$$

$$= 100062 \text{ N/m}^2$$

Pressure at a point, which is at a depth of 3 m from the free surface of the liquid is given by,

$$p = \rho_1 \times g \times Z_1$$

$$= (1.53 \times 1000) \times 9.81 \times 3 = 45028 \text{ N/m}^2$$

∴ Gauge pressure, p = **45028 N/m^2. Ans.**

Now absolute pressure = Gauge pressure + Atmospheric pressure

= 45028 + 100062 = **145090 N/m^2. Ans.**

- **CL203 FLUID MECHANICS**
- **FLUID STATICS:**

- **<u>Reference:</u>**
- Fluid Mechanics by Fox
- Fluid Mechanics by Bansal
- NPTEL