Optimum Design and Design Strategy

ok : *Plant Design and Economics for Chemical Engineers*, M.S. Peters and K. D. Timmerh

Chapter 11 (4th Edition)

An optimum design is based on the best or most favourable conditions

In engineering process design, the criterion for optimality can most often be reduced a consideration of costs and profits

The optimum for a process design is the most cost-effective selection, arrangement, sequencing of process equipment and operating conditions for design

Steps in the development of an optimum design:

- Determine which factor needs to be optimized, i.e., determine the objective function
- The process variables and constraints that affect the objective function needs to be identified and relationships developed to show how the variables affect the chosen function (process variables are variables that affect the values of the objective function while constraints are limitations on the process)
- Finally, these relationships are combined graphically or analytically to give the desired optimum conditions

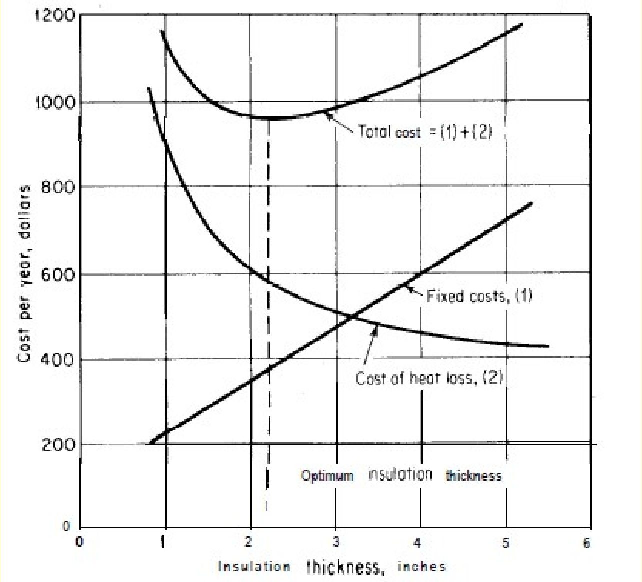
An optimum economic design could be based on conditions giving *least cost per unit time* or *maximum profit per unit production*

When one design variable is changed, often some costs increase and others decrease. Under these conditions, the total cost may go through a minimum and at one value of the particular design variable and this value is the optimum value of the variable.

An example illustrating the principles of an optimum economic design is shown in the Figure. The problem here is to determine the optimum thickness of insulation

The annual fixed costs increases as the insulation thickness is increased, while the operating costs decrease as the cost of heat loss decreases.

The sum of the costs goes through a minimum at the optimum insulation thickness.



Though cost considerations and economic balances are the basis of most optimum designs, there are times when factors other than cost determine the most favourable conditions such as optimum operating temperature for a reactor based on equilibrium and reaction rate limitations. Optimum temperature may be based on maximum percentage conversion or maximum amount of final product per unit time

Optimization procedure with one variable

There are many cases in which the factor being optimized is a function of a single variable.

For eg., obtain the insulation thickness which gives the least total cost (figure given earlier)
The primary variable involved is the thickness of the insulation, and relationships can be developed showing how this variable affects all costs

The fixed charges have a direct relationship with insulation thickness and can be written as

$$Fixed\ charges = \Phi(x) = ax + b \qquad x = insulation\ thickness$$

The cost of heat lost as a function of insulation thickness can be obtained from data on the value of steam, properties of the insulation, and heat-transfer considerations. All other costs, such as maintenance and plant expenses, can be assumed to be independent of the insulation thickness

$$Cost\ of\ heat\ loss = \Phi^i(x) = \frac{c}{x} + d \qquad a, b, c, d = positive\ constants$$

$$Total\ variable\ cost = C_T = \Phi(x) + \Phi^i(x) = ax + b + \frac{c}{x} + d$$

The graphical method for solving this optimization problem was shown earlier (slide 3). The optimum thickness of insulation is found at the minimum point on the curve obtained by plotting total variable cost versus insulation thickness

For the analytical method, the slope of the total variable cost curve is zero at the point of optimum insulation thickness

$$Total\ variable\ cost\ = C_T = \Phi(x) + \Phi^i(x) = ax + b + \frac{c}{x} + d$$

$$\frac{dC_T}{dx} = a - \frac{c}{x^2} = 0$$

$$x^2 = \frac{c}{a}$$

r $$x = \left(\frac{c}{a}\right)^{1/2}$$

o check whether it is the maximum or the minimum point,

$$\frac{d^2C_T}{dx^2} = \frac{2c}{x^3}$$

C is positive, and x being positive, the second derivative > 0

ence, at the point of inflection (value of x), the total variable cost is minimum

Optimization procedure with two or more variables

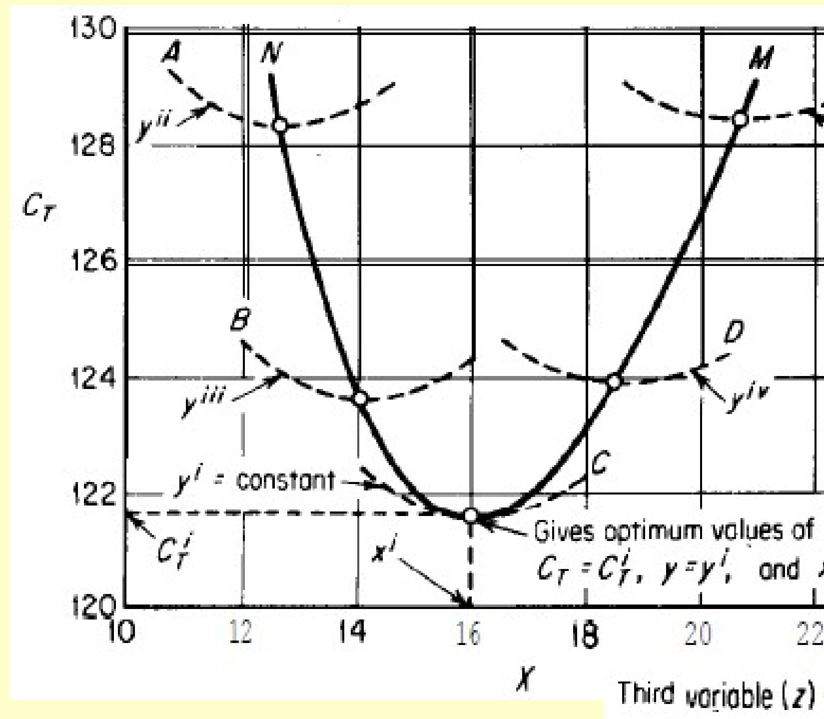
hen two or more variables affect the factor being optimized, the approach is similar to the method llowed for a single variable

$$C_T = f(x, y)$$

e function may be of the form

$$C_T = ax + \frac{b}{xy} + cy + d \qquad a, b, c, d = positive\ constants$$

the graphical method, the relationship between C_T , x, y could shown as a curved surface on the three-dimensional (3-D) plot, t a 3-D plot may not be practical for several engineering terminations



tead a 2-D plot can be used to depict the problem. Here, the tor being optimized (C_T) is plotted versus one of the independent iables (x) while the second variable (y) is held constant

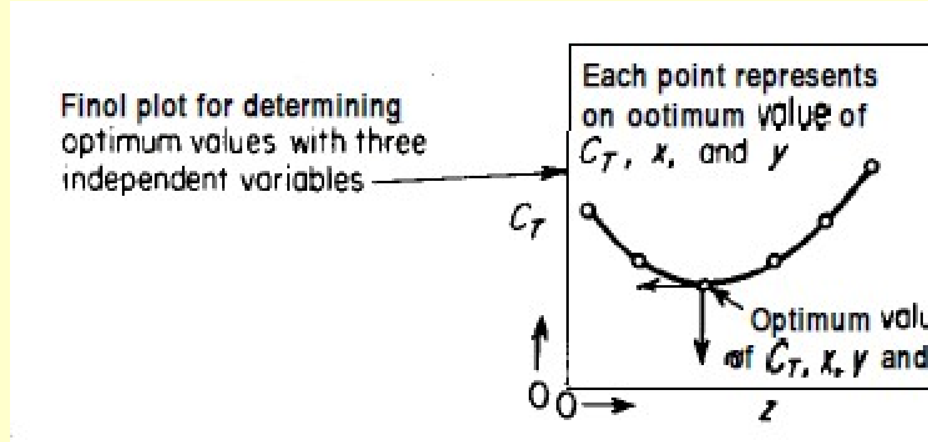
eries of such plots at different constant values of y are plotted as tted lines. Each of these plots (A, B, C, D, E, F) gives one value of variable x at the point where the total cost is minimum

e curve NM is the locus of all these minimum points and the timum value of x and y occurs at the minimum point on curve M

If $C_T = f(x,y,z)$, then plots such as the previous one is drawn for different values of z

Each plot gives an optimum value of x, y and C_T for a particular z

These are then plotted versus z to get the overall optimum values of x, y, z and C_T



Analytically, the estimation can be done by differentiating the function with respect to one variable keep
he others constant, for each variable

The function is of the form $C_T = ax + \frac{b}{xy} + cy + d$

$$\frac{\partial C_T}{\partial x} = a - \frac{b}{x^2 y} \quad \text{and} \quad \frac{\partial C_T}{\partial y} = c - \frac{b}{xy^2}$$

At the optimum condition, $\frac{\partial C_T}{\partial x} = 0$ and $\frac{\partial C_T}{\partial y} = 0$

Therefore, $x^2 y = \frac{b}{a}$ and $xy^2 = \frac{b}{c}$

From the first expression we get, $y = \frac{b}{a\,x^2}$

Replacing this in the second equation we get, $x\left(\frac{b}{a\,x^2}\right)^2 = \frac{b}{c}$

$$x\left(\frac{b}{a\,x^2}\right)^2 = \frac{b}{c} \qquad \text{or} \qquad \frac{x b^2}{a^2 x^4} = \frac{b}{c} \qquad \text{giving} \quad x^3 = \frac{cb}{a^2}$$

Therefore, $$x = \left(\frac{cb}{a^2}\right)^{1/3}$$

nd $$y = \left[\frac{b}{a}\left(\frac{a^2}{cb}\right)^{2/3}\right] = \frac{a^{1/3} b^{1/3}}{c^{2/3}}$$

$$y = \left(\frac{ab}{c^2}\right)^{1/3}$$

xample: The cost of two independent process variables f_1 and f_2 affects the total cost, C_T (in lakhs of rup
f the process as per the following function

$$C_T = 100f_1 + \frac{1000}{f_1 f_2} + 20f_2^2 + 50$$

Vhat is the lowest total cost in lakhs of rupees? *[GATE 2015]*

otal cost, $$C_T = 100f_1 + \frac{1000}{f_1 f_2} + 20f_2^2 + 50$$

or lowest total cost, $$\frac{\partial C_T}{\partial f_1} = 0 \quad \text{and} \quad \frac{\partial C_T}{\partial f_2} = 0$$

$\frac{T}{1} = 100 - \frac{1000}{f_1^2 f_2} = 0 \quad \rightarrow \quad f_1^2 f_2 = \frac{1000}{100} = 10$ .... (1)

$\frac{T}{2} = 40 f_2 - \frac{1000}{f_1 f_2^2} = 0 \quad \rightarrow \quad f_1 f_2^3 = \frac{1000}{40} = 25$ ....(2)

om (2) $f_1 = \frac{25}{f_2^3}$

eplacing in (1) we get, $\left(\frac{25}{f_2^3}\right)^2 f_2 = 10$

, $f_2^5 = \frac{25 \times 25}{10} \quad \rightarrow \quad f_2 = 2.286$

ow, $f_1 = \frac{25}{f_2^3} = \frac{25}{(2.286)^3} \quad \rightarrow \quad f_1 = 2.093$

tal cost, $C_T = 100 f_1 + \frac{1000}{f_1 f_2} + 20 f_2^2 + 50$

$$C_T = 100(2.093) + \frac{1000}{2.093 \times 2.286} + 20(2.286)^2 + 50$$

$$= Rs.\, 572.82$$

Break-even chart for production schedule and its significance for optimum analysis

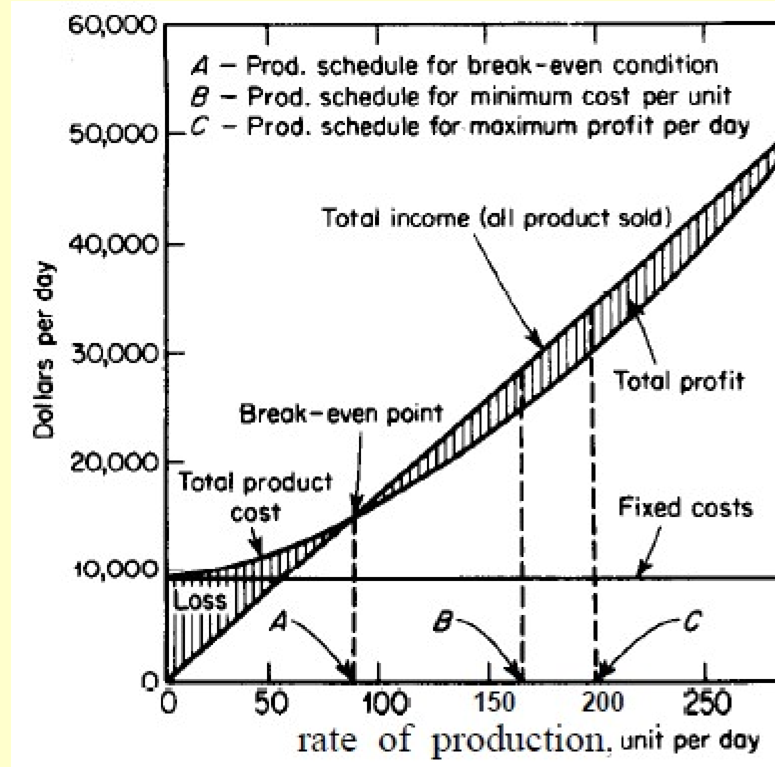
One of the factors that has an important effect on the overall costs or profits in a plant operation is the fraction of total available time during which the plant is in operation.

If the plant stands idle or operates at low capacity, costs for raw materials and labour are reduced, but costs for depreciation and maintenance continue at essentially the same rate though the plant is not in full use

There is a close relationship among operating time, rate of production and selling price

It is desirable to operate the plant at a schedule which will permit maximum utilization of fixed costs while simultaneously meeting market sales demand and using the capacity of the plant production to give the best economic results

The figure shows how production rate affects costs and profits



The fixed costs remain constant while the total product cost and profit increases with increased rate of production. The point where total product cost equals total income represents the break-even point

The optimum production schedule must be at a production rate higher than that corresponding to the brea point.

ptimum Production Rates in Plant Operation

The same principles used for developing an optimum design can be applied when determining the most favourable conditions in the operation of a manufacturing plant

The optimum rate of production can be determined from the analysis of the costs involved and other factors affecting plant operation

The optimization can be done based on

- ✓ Total product cost per unit time
- ✓ Total profit per unit time

Variables involved is the production rate or the amount of product produced per unit time

otal product cost per unit time may be divided into the following –

(i) *Operating costs* – depends on rate of production and includes labour, raw materials, power, eat etc and depends on the amount of material produced

(ii) *Organizational costs* – independent of rate of production and includes supervisory, ngineering, administrative personnel, physical equipment and other services or facilities which nust be maintained irrespective of amount of material produced

Operating costs are often considered on the basis of one unit of production

In this case, the operating costs may be divided into two types:

(a) Minimum expenses for raw materials, labour, power that must be paid for each unit of production as long as any amount of material is produced

(b) Extra expenses due to increasing rate of production. This is called "super-production costs". Examples of super-production costs are extra expenses caused by overload on power facilities, additional labour or decreased efficiency of conversion

per-production costs per unit of production = mP^n

where, P : production rate (units of production/time) ; m, n : constants

ppose, h = operating cost (remains constant per unit of production

O_c = organization cost per unit of time

en,

c_T = total product cost per unit of production

$$c_T = \left(h + mP^n + \frac{O_c}{P}\right)$$

Total product cost per unit time $(C_T) = c_T P$

$$C_T = \left(h + mP^n + \frac{O_c}{P}\right)P$$

II) Profit per unit of production, $r = s - c_T$

$$r = s - \left(h + mP^n + \frac{O_c}{P}\right)$$

s = selling price per unit product

V) Profit per unit of time, $R' = rP$

$$R' = \left[s - \left(h + mP^n + \frac{O_c}{P}\right)\right]P$$

A) Optimum production rate for minimum cost per unit of production

ften it is necessary to know the rate of production that would give the least cost of one unit of material roduced. This information shows the selling price at which the company would be forced to cease perations or else will operate at a loss

ow, $Total\ product\ cost\ per\ unit\ of\ production = c_T = \left(h + mP^n + \frac{O_c}{P}\right)$

or optimum production rate, $\frac{dc_T}{dP} = 0$

plot of total product cost per unit of production versus production rate shows a minimum)

$$\frac{dc_T}{dP} = mnP_O^{n-1} - \frac{O_c}{P_O^2} = 0$$

or, $mnP_O^{n-1} = \frac{O_c}{P_O^2}$

or, $$\boldsymbol{P_o = \left(\frac{O_c}{mn}\right)^{\frac{1}{n+1}}}$$

The **optimum production rate (P_o)** gives the maximum profit per unit of production if the selling price remains constant

B) Optimum production rate for maximum total profit per unit time

n most business ventures, the amount of money earned (profit) over a given period of time is more importan
han the amount of money earned for each unit of product sold

rofit per unit time, $R' = \left(s - h + mP^n + \frac{O_c}{P}\right)P$

or optimum production rate, $\frac{dR'}{dP} = 0$

plot of profit per unit time versus production rate goes through a maxima)

$$\frac{dR'}{dP} = \frac{d}{dP}\left[sP - hP - mP^{n+1} - O_c\right] = 0$$

or, $s - h - m(n+1)P_o^n = 0$

or, $s - h = m(n+1)P_o^n$

or, $P_o^n = \frac{(s-h)}{m(n+1)}$

or, $$P_o = \left[\frac{(s-h)}{m(n+1)}\right]^{1/n}$$

he **optimum production rate (P_o)** gives the maximum profit per unit of time if the selling price remains const

ample:

A plant produces refrigerators at the rate of P units per day. The variable costs per refrigerator has been foun to be \$ $47.73 + 0.1\ P^{1.2}$. The total daily fixed charges are \$1750, and all other expenses are constant at \$7325 per day. If the selling price per refrigerator is \$173, determine:

a) The daily profit at a production schedule giving the minimum cost per refrigerator.

b) The daily profit at a production schedule giving the maximum daily profit.

c) Production schedule at the break even point

Total cost per refrigerator = $c_T = 47.73 + 0.1\ P^{1.2} + \frac{(1750+7325)}{P}$

For a production rate giving minimum cost per refrigerator, $\frac{dc_T}{dP} = 0 = 0.1 \times 1.2 P_O^{0.2} - \frac{(9075)}{P_O^2}$

$$0.12\ P_O^{0.2} = \frac{(9075)}{P_O^2}$$

$$P_O^{2.2} = \frac{(9075)}{0.12} = 75625$$

P_O = 165.03 ~ 165 refrigerators/ day

Daily profit at this production rate

$$R' = rP = \left(s - h - mP^n - \frac{O_c}{P}\right) P = \left[173 - 47.73 - 0.1(165)^{1.2} - \left(\frac{9075}{165}\right) 165\right]$$

$$= \$\ 4035.54 \sim \$\ 4036$$

b) Daily profit = $R' = \left(173 - 47.73 - 0.1\,P^{1.2} - \frac{9075}{P}\right)P$

For maximising profits per day, $\frac{dR'}{dP} = 0 = 173 - 47.73 - 0.1\times 2.2\,P_o^{1.2}$

or, $2.2\,P_o^{1.2} = 125.27$

or, $P_o = 197.78 \sim 198\ units/day$

Daily profit = $\left[173 - 47.73 - 0.1\,(198)^{1.2} - \frac{9075}{198}\right]198$

$= \$\,4439.25 \sim \$\,4440$

c) At the break-even point, the total product cost = total income

The optimum production schedule must be a rate higher than that corresponding to the break-even point

$$Total\ S.P. = \left(h + mP^n + \frac{O_c}{P}\right)P$$

$$s = h + mP^n + \frac{O_c}{P}$$ (based on one unit of product)

$$173 = 47.73 - 0.1\,P^{1.2} - \frac{9075}{P}$$

Solving by trial and error, $P = 88\frac{units}{day}$ at break-even point