Department of Mathematics
Birla Institute of Technology,
Mesra, Ranchi-835215

Syllabus for Pre PhD Qualifying Exam

Module - 1

Algebra: Vector spaces, subspaces, linear dependence, basis and dimension, Linear transformation, range space, null space, rank and nullity, Matrix representation of a linear transformation. Change of basis, Eigenvalues and eigenvectors, Inner product, orthogonality, Gram-Schmidt process, orthogonal expansion, Quadratic forms, reduction to normal form.

Module - 2

Complex Analysis: Complex Integration, Cauchy's Integral Theorem, Cauchy's Integral Formula, Morera's Theorem, Liouville's Theorem, Expansion of functions of complex variable in Taylor's and Laurent's series, singularities and poles, Cauchy's Residue Theorem, contour integration, Conformal mapping and applications, bilinear transformation, Schwartz - Christoffel transformation.

Module - 3

Ordinary Differential Equations: Systems of first order equations: linear systems, homogeneous linear systems with constant coefficients, series solution using Frobenius method, existence, uniqueness, and theory of stability.

Special Functions: Solution of differential equation in series: Bessel, Legendre and Hypergeometric differential equations and their solutions, Bessel's function of different kinds, integral representation of Bessel's functions, orthogonality of Bessel's functions, modified Bessel's functions, Legendre polynomials, Rodrigue's formula, orthogonality of Legendre's polynomials, associated Legendre's function, Hypergeometric function and its integral representation.

Partial Differential Equations: Classification of partial differential equations of second order, method of separation of variables, solution of one - dimensional heat and wave equation by method of separation of variables, Homogeneous linear partial differential equations with constant coefficients, non - linear partial differential equations of second order.

Module - 4

Fourier Series: Periodic functions, Dirichlet conditions, Euler's formula, half - range series, Fourier series representation of functions of arbitrary period.

Integral Transforms: Laplace transform and its inversion, convolution theorem, Fourier transform and its inversion, Fourier sine and cosine transform, convolution, Fourier transform of derivatives.

Sturm Liouville's Problem: Sturm Liouville's differential equation and its solution, eigenvalues, eigen functions, expansion.

Module - 5

Discrete Mathematics: Algebra of sets, introduction to combinatory, counting techniques, principle of inclusion - exclusion, the pigeon - hole principle and its application, recurrence relation, generating function, mathematical logic, propositional calculus, basic logical operation, truth tables, Functions, composition of functions, functions for computer science, relations, equivalence relation, digraphs, computer representation of relations, Warshall's algorithm, Algebraic structures: group, semi - group, cyclic group.

Graph Theory: Graphs, types of graphs, properties and applications of graphs, matrix representation of graphs, Euler and Hamiltonian graphs, isomorphic graphs, representation of graphs in computer memory, Trees, properties of trees, pendant vertices in a tree, spanning trees, fundamental circuits, spanning trees in a weighted graph.

Module - 6

Hydrodynamics: Compressible and incompressible Navier - Stokes equations, stream functions, vorticity method for incompressible flow, exact solutions of Navier - Stokes equations (Coutte flow, flow between two concentric cylinders, stagnation point flow), Boundary layer equations for two - dimensional incompressible flow, boundary layer along a flat plate, expressions for boundary layer along a flat plate, expressions for boundary layer thickness, displacement thickness, momentum thickness and similar solutions.

Module - 7

Numerical Methods: Approximation of functions, their derivatives and integrals by interpolation, Finite and divided differences, Iterative methods for solving nonlinear and linear equations, convergence, Power method for largest eigenvalue, Numerical Solution of ordinary differential equations, Initial value problems by Runge - Kutta and predictor-corrector methods, Boundary value problems by finite difference methods and method of weighted residuals, Numerical Solution of Laplace and Poisson equations.

Module - 8

Probability and Statistics: Sample space, events and probability axioms, Random variable, probability density function, mathematical expectation, variance, Probability distributions: Binomial, Normal and Poisson distributions, Sampling Distributions: Testing of Hypothesis, Tests of significances: Chi - square test, t – test and F – test.

Module - 9

Theory of Elasticity: Theory of stress, types of external forces, the stress tensor, equations of motion and equilibrium in terms of the components of the stress tensor, Generalized Hooke's law, various cases of elastic symmetry of body, complete system of fundamental equation in the theory of elasticity, simple problems of the theory of elasticity, the plane problems in the theory of elasticity, Interior of the earth, wave motion, the body waves, surface waves, propagation of elastic waves in different media, Love waves and Rayleigh waves.

Module - 10

Operations Research: Linear programming problems, convex set, convex functions, Simplex method and its variants, duality, sensitivity analysis, Transportation problems, initial basic feasible solution and optimal solution, degeneracy, Assignment problems, applications of TP and AP, Nonlinear programming problems, Kuhn-Tucker conditions.